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ACCOUNT

OF THE

MAGNETICAL OBSERVATORY OF DUBLIN,

AND OF THE

INSTRUMENTS AND METHODS OF OBSERVATION

EMPLOYED THERE.

BY

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INTRODUCTION.

To the labours of Englishmen we are indebted for the important discovery, that the direction of the earth's magnetic force at a given place is not constant; that the angles by which it is determined, the declination and inclination, are subject to variations, both of a *periodical* and of a *progressive* kind; and that the periodical variations are dependent as well on the *hour* as on the *season*. From the first discovery of the phenomenon by **Graham**, in 1722, the hourly changes of the declination were much studied. A close approximation to their empirical laws was furnished by the observations of **Canton**, which commenced about the year 1756. These observations were followed by those of **Cassini**, at Paris, in the year 1783; and the examination of the subject was resumed in the early part of the present century, by **Humboldt** at Berlin, by **Beaufoy** in England, and by **Arago** in France.

Of these later researches, those of M. **Arago** are of especial value, on account of their completeness, continuity, and extent. The observations, which commenced about the year 1818, have not yet been given to the world in detail; but among their results may be mentioned the important fact of the invariable connexion of the *Aurora Borealis* with the greater perturbations of the needle. In order further to elucidate these phenomena, M. **Arago** requested M. **Kupffer** to undertake a series of corresponding observations of the magnetic declination at Kazan. These observations were made in the years 1825 and

1826; and their comparison revealed the remarkable fact, that the irregular movements of the needle occurred at the same time at the two places, although they differed from one another by more than 47 degrees of longitude. As yet, however, the similarity in the course of the movements was undetected.

The first complete system of corresponding observations was that organized by HUMBOLDT in the year 1829, with the assistance of M. KUPFFER. Simultaneous observations of declination changes were taken at Berlin and Freiberg seven times in the year, the observations being continued at intervals of an hour for forty-four hours; and the Imperial Academy of Russia adopting the project with zeal, the chain of magnetic stations was extended across the whole of Northern Asia. Observatories were erected at Petersburg and at Kazan; and corresponding observations were made at Moscow, at Nicolajeff, at Barnaoul and Nertchinsk, in Siberia, at Sitka, and even at Pekin. A comparison of the results obtained at four of these stations, made by HUMBOLDT and DOVE in the year 1830, showed that the changes were not only synchronous, but that moreover their course was in general parallel.

The next important step in the advancement of the science of terrestrial magnetism by means of fixed observatories was made by GAUSS. Having contrived instruments, and methods of observation, far more perfect and exact than any before employed, he proceeded to apply them in observing the simultaneous changes of the magnetic declination at distant places. At the very commencement of this inquiry he ascertained, that the direction of the resultant of the forces acting on the horizontal needle was continually changing; that every such change, however minute, at any one place of observation had its counterpart at every other; and that the curves which represented the course of these changes were parallel. In order to determine, therefore, the actual character of these changes, it was necessary that the observations should be made at very short intervals; and GAUSS was thus led to organize a cooperative system, in which the observations were taken simultaneously at intervals of five minutes. These were carried on through twenty-four hours, six times in the year; and magnetic stations taking part in the system were established at Altona, Augsburg, Berlin, Breda, Breslau, Copenhagen, Dublin, Freiberg, Gottingen, Greenwich, Hanover, Leipsic, Marburg, Milan, Munich, Petersburg, Stockholm, and Upsala. This plan of combined observation commenced in the year 1835; and the inventive mind of GAUSS soon after supplied the means of observing the changes of the horizontal component of the magnetic intensity, with an exactness equal to that already applied to the declination.

The subject of the perturbations of the magnetic declination has been here dwelt upon, because it is from the interest attached to these phenomena that the establishment of most of the observatories for magnetical research has taken its rise. Those who took part in these observations perceived, from the very first, that the cause, whose effects they were engaged in unravelling, was one of very wide agency; and that if its physical nature could by such means be determined, that of the more stable forces engaged in the phenomena of terrestrial magnetism might be expected to appear also. It was naturally supposed likewise, that the investigation of these remarkable phenomena would shed a light upon other hidden portions of the *physique du globe*. "That which characterizes our epoch," says Baron HUMBOLDT, in his letter to the President of the Royal Society, in 1836, "at a time distinguished by grand discoveries in optics, electricity, and magnetism, is the possibility of connecting phenomena by the generalization of empirical laws, and the mutual aid afforded by sciences which had long remained isolated. At the present day, simple observations upon horary declination, or magnetic intensity, made simultaneously in situations very distant from each other, reveal, so to speak, what passes at profound depths in the interior of our planet, and in the superior regions of the atmosphere."

But important as these phenomena undoubtedly are, they form but a small part of the proper business of a magnetical observatory. The interest which surrounded the new subject diverted attention from the periodical and secular changes, which had, as regards the declination at least, occupied the attention of the early observers; and yet the latter phenomena are not only as important in themselves, but also, from the long and uninterrupted series of observations which are required for the development of their laws, they stand, more than any other class, in need of the appliances of a regular observatory. In order to embrace *all* that relates to the earth's magnetic force at a given place, the *absolute* values of the three elements which determine its direction and intensity must be accurately observed at given epochs, and the law of their variations, both *secular* and *periodic*, traced by the help of an uniform and continuous system of observation; lastly, the knowledge of the causes engaged in producing the *irregular* fluctuations is to be promoted by observations made in correspondence with other observatories at fixed epochs, or whenever the marked nature of the change may have arrested the attention of the observers.

The magnetical observatory of Paris, under the direction of M. Arago, was probably the first at which any complete system of observation, bearing upon the periodical variations, was regularly carried on. The Russian observatories, under the direction of M. Kupffer, were the next to prosecute the subject, and observations were regularly made, eight times a day, commencing in the year 1834, at Petersburg, Catharineburg, Barnaoul, and Nertchinsk; meteorological observations were likewise made, at these and other stations in Russia, at the same hours. Lastly, the example thus set was followed in the Milan observatory, under the direction of MM. Carlini and Kreil, the observations being taken there six times daily, commencing in the year 1836; and M. Kreil has continued the same system on an extended scale at Prague, to which place he has since removed.

Being deeply impressed with the importance of such researches, conducted upon an extended plan, I ventured, in the spring of the year 1837, to urge upon the heads of the University of Dublin the scientific advantages which might be expected to arise from the establishment of a complete observatory in connexion with the University, in which all the researches connected with the sciences of terrestrial magnetism and meteorology might be systematically conducted. The governing body of the College entered into the project with a promptitude and zeal which, I may be permitted to say, they have ever shown where the real interests of science were concerned; the requisite sum was immediately voted, and the building was commenced in the summer of the same year. Its progress, however, was slow, partly on account of the care demanded in its construction, and partly from other accidental causes; and it was not completed until the spring of the following year. In the month of July, 1838, some of the instruments were mounted; and the regular course of observation commenced, under my direction, in the November following.

At the period now referred to, this observatory, and that at Greenwich under the direction of the Astronomer Royal, built about the same time, were the only establishments specifically directed to magnetical research, in connexion with Britain. Since that time this country has nobly reassumed the place which, in reference to this branch of science, she had nearly abandoned to her continental neighbours. At the instigation of the British Association, and of the Royal Society, four complete magnetical observatories have been established by the Government, at Toronto, St. Helena, the Cape of Good Hope, and Van Diemen's Land(*a*). The munificence of the Directors of the East India Company has founded and furnished an equal number, at Simla, Madras, Sin-

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(*a*) The observations made at these observatories are now preparing for publication under the superintendence of Colonel SABINE, assisted by Lieutenant RIDDELL, having previously passed through my hands for examination and correction.

gapore, and Bombay; and Sir THOMAS BRISBANE has since added another to the list, at his own expense, at Kelso in Scotland. These observatories, as well as the moveable observatories of the Antarctic expedition, are all provided with instruments precisely similar to those of the Dublin magnetical observatory; and their directors, Lieutenants RIDDELL, LEFROY, and WILMOT, Captain BOILEAU, Lieutenants LUDLOW and ELLIOTT, and Mr. RUSSELL, have learned in Dublin the methods of observation.

The distinguishing characteristic of this undertaking,—that which gives it unity and greatness,—is, that the same plan of observation is followed out in all these distant stations, by observations strictly simultaneous, made according to the same instrumental methods, and with the same instrumental means. In order to give a still wider extension to this scheme, which was matured under its direction, the Royal Society solicited the cooperation of foreign states. The invitation was responded to in a spirit unparalleled in the history of science. Most of the foreign observatories were reorganized, on a scale of greater completeness; many new ones were added; and thirty-three observatories are now in operation, following out the same plan in all its details. Of these eleven (including Greenwich and Dublin) are established in Britain and her dependencies; and ten have been founded and equipped by the Russian Government, viz., at Petersburg, Catharineburg, and Kazan, in Russia proper; at Helsingfors, in Finland; at Nicolajeff, in the Crimea; at Tiflis, in Georgia; at Barnaoul and Nertchinsk, in Siberia; at Sitka, in North America, and at Pekin in China. Of the rest, one has been established by the French Government, at Algiers; one by the Belgian, at Brussels; two by Austria, at Prague and Milan; one by Prussia, at Breslau; one by the Bavarian Government, at Munich; and one by the Spanish, at Cadiz; there are two in the United States, at Philadelphia and Cambridge; one at Cairo, founded by the Pacha of Egypt; one at Trevandrum, in India, by the Rajah of Travancore; and one by the

King of Oude, at Lucknow. The observatories of Brussels, Breslau, Cadiz, Cambridge, Algiers, Cairo, Trevandrum, and Lucknow, are provided with instruments similar to those of Dublin.

In devising the instruments, I have been, of course, largely indebted to the preceding labours of GAUSS, whose improvements in the construction and use of magnetical instruments have given to their results all the precision of astronomical measurements. These improvements may be reduced to two principal heads, viz., first, the use of massive magnetized bars in place of light needles, and, secondly, the method of observing at a distance from the instrument. To the former of these we owe the diminished effect of currents of air, and other accidental influences, upon the position of the magnetic bar; to the latter the removal of those disturbing causes which are connected with the near approach of the observer's person. I have carefully adhered to both of these principles, though with considerable modification in their application. With respect to the first, it is to be observed, that although the accuracy of observation is increased, as regards the effect of disturbing causes, by increasing the size of the bar, it is, on the other hand, lessened by the augmented time of its vibration. It is well known, that in order to eliminate the effect of the vibratory movement, it is necessary to take three or more readings of the position of the bar, the interval between the first and last being, at least, equal to the time of a double oscillation. Now, the exactness of this method rests upon the assumption, that the actual changes in the *mean* position of the magnet may be regarded as *uniform* during the time of observation; and this assumption, it is evident, will be nearer to the truth, as the time of observation (and, therefore, the time of vibration) is shorter. There is, consequently, an obvious limit to the most advantageous size of the bar. In periods of disturbance, the changes of mean position which take place in the time of a double oscillation of GAUSS's large bars, are sometimes very far from being uniform; and cases even occur, where the error in the deduced position, arising from this source, is greater than that due to the vibratory movement, which it is the object of this method of observation to remove. There are other advantages, of a practical kind, belonging to smaller bars; the chief of which is the very important one, that all the instruments may be placed in one room, of moderate dimensions, without incurring a disturbing action exceeding the limits of a small correction. Guided by these considerations, and especially the latter, I have thought it advisable to employ magnetic bars of a size considerably smaller than that recommended by GAUSS.

The second practical principle, namely, the removal of the observer's person from the neighbourhood of the instrument, has been attained by GAUSS, by determining the position of the magnet by means of an attached mirror which reflects the divisions of a distant scale. For this method of reading, I have substituted that furnished by the principle of the collimator, the lens and scale being both attached to the magnet. The advantages of this method of reading seem to be, first, that the scale (being fixed to the magnet itself) is not liable to derangement; and, secondly, that the illumination being as abundant as required, the apertures in the box may be closed with glass, and thus the danger of currents of air much lessened. I may add, that a single wax candle, in the middle of the room, is sufficient for the illumination of the three instruments, and that thus the disturbing effects of unequal temperature are still further obviated.

I have thought it necessary to explain, in detail, the grounds upon which I have been induced to deviate from the plan of the instruments devised by GAUSS, for the observation of the magnetic declination and horizontal component of the magnetic force, because these instruments exhibit in the details of their construction much of that careful consideration which characterizes all the works of that illustrious mathematician, and are justly stamped with the authority of his high name. A third instrument, however, is required for the observation of the remaining element. This want has been in part supplied by MM. KREIL and KUPFFER, in the instruments which they have respectively devised for observing the changes of the magnetic inclination. The means which I have employed in determining the variations of the third element will be described in the following pages.

The present publication, the expenses of which have been provided for by the liberality of the Board of Trinity College, will contain the results of observation in full detail, preceded by an account of the observatory itself, and of the instruments and methods of observation employed. The observations of the first two months, being less complete than those which follow, are not included; so that the regular series of the published observations commences in January, 1839. During the months of January and February the observations were taken eight times daily, namely, at the hours 9 A.M., 10, 11, 12, 1 P.M., 2, 3, 5; and in March, and the three following months, the hours 7 A.M., 7 P.M. and 9 P.M. were added. In the middle of the month of June the observations were interrupted, in order that the instruments might be used by the directors of the observatories then about to be established by the Government, in learning the details of their manipulation. They were resumed in August, the observations being taken eight times daily, namely, at the alternate hours from 7 A.M. to 9 P.M., inclusive; and at the beginning of the year 1840, the number of assistant observers was increased to three, and the observations were made *every alternate hour* during the day and night, according to the enlarged plan of combined observation already referred to.

MAGNETICAL OBSERVATORY OF DUBLIN.

THE Magnetical Observatory of Dublin is situated in an open space in the gardens attached to Trinity College, and at a distance of 160 feet from the nearest building. Its approximate position is in latitude 53° 21′ North, and longitude 6° 16′ West. The dimensions of the building are forty feet in length, and thirty in depth(*a*). (Plate I). It is constructed of Portland stone, the interior stone of the building being the argillaceous limestone (*calpe*) of the valley of Dublin; several specimens of each of these stones had been previously submitted to a rigid examination, and found to be entirely devoid of any magnetic influence. The walls are studded internally, for the purpose of maintaining a uniform temperature, as well as to protect from damp. The nails employed in the woodwork are of copper, and the other metallic fastenings (locks, hinges, &c.) of brass, no iron whatever being used in any part of the structure.

The interior is divided into one principal room and two smaller rooms; one of the latter serving as a closet, and the other as a vestibule (fig. 2). The principal room is thirty-six feet long, and sixteen feet wide; and it has projections in its longer sides, which increase the breadth of the central part to twenty feet. The medial line of this room forms an angle of four degrees with the meridian, to the east of north(*b*). It is lighted by a dome light at top, and by two windows, one at the northern and the other at the southern end.

(*a*) The building was erected under the superintendence, and after the designs of Frederick Darley, Esq. Architect. It is of the Doric order, the front elevation being partly copied from an Athenian model (fig. 1).

(*b*) This direction was not chosen with reference to any magnetic purpose; but merely in order to accommodate the principal lines of the building to those of the neighbouring ones, it being situated in ornamental ground.

There are six pillars of Portland stone in this room, for the support of the instruments. These are imbedded in solid masonry beneath the floor; and the supports of the floor are framed around them, so that they are completely insulated. The three pillars which support the magnetical instruments, A, B, C, (fig. 2) are placed, one near the centre of the east wall, and the other two near the ends of the wall opposite; the distance of the centres of the pillars from the walls is three feet, and their mutual distances are AB = 30 feet, AC = BC = 19 feet. The height of these pillars above the floor is four feet; and their diameter twenty inches. The pillars D, D, close to the southern window, support the transit instrument; and the pillar E, near the opposite end of the room, the theodolite. The axis of this latter pillar is in the line in which the magnetic meridian passing through the axis of the pillar A, intersects the meridian of the transit, the distance AE being five feet. To provide for the change of position of this pillar, with the alteration of the declination in the course of time, there is a low stone wall beneath the floor, nine feet long and three feet wide, the middle line of which is in the meridian of the transit; on this is placed a massive square base for the pillar, the position of which may be altered hereafter when required, provision having been made in the flooring for this removal.

In addition to this building, and at a considerable distance from it, a separate room has recently been constructed, for absolute measures of the inclination and intensity. The displacement of the instruments in the principal observatory, their readjustment, and the consequent interruptions of the regular series of observation, will thereby be avoided. This pavilion is twelve feet square in the interior. For the purpose of preserving a uniform temperature, the walls are studded, and the door is double. The room is lighted by a single window at top, which is provided with a blind, running on horizontal rods, for the purpose of excluding the direct rays of the sun. There are three granite pillars, resting on solid masonry beneath the floor, for the support of the instruments; the lines connecting the centres of these pillars form an equilateral triangle, whose sides are six feet in length, the direction of one of them being parallel to the magnetic meridian.

In order to know all that relates to the earth's magnetic force at a given place, observation must furnish the values of three elements. Those which naturally present themselves for immediate determination are, the *intensity* of the force itself, and the two angles (the *declination* and *inclination*) which determine its direction. A vertical plane being conceived to pass through the direction of the force, that direction will be determined when its inclination to the horizon is given, as well as the angle which the plane itself forms with the meridian; and if, in addition to these quantities, we likewise know the number which expresses the ratio of the intensity of the force to some established unit, it is manifest that the force is completely determined. We may substitute for these, however, any other system of elements which are connected with them by known relations. Thus, the force being resolved into two portions in the plane of the magnetic meridian, one of them *horizontal* and the other *vertical*, it is manifest that these two components may be substituted for the total intensity and the inclination,—or either of the former may be taken in lieu of either of the latter,—the considerations which should govern our choice being the exactness of the observed results, and the facility of their determination.

The relations between these elements and their changes are readily obtained. If R denote the force, X and Y its horizontal and vertical components, and θ the inclination, we have

$$X = R\cos\theta; \quad (1) \qquad Y = R\sin\theta. \quad (2)$$

Differentiating, and dividing by the equations themselves, we find

$$\frac{\Delta X}{X} = \frac{\Delta R}{R} - \tan\theta . \Delta\theta; \tag{3}$$

$$\frac{\Delta Y}{Y} = \frac{\Delta R}{R} + \cot\theta . \Delta\theta. \tag{4}$$

Whence we have, by elimination,

$$\Delta\theta = \sin\theta\cos\theta \; \frac{\Delta Y}{Y} - \frac{\Delta X}{X}\Big); \tag{5}$$

$$\frac{\Delta R}{R} = \cos^2\theta . \frac{\Delta X}{X} + \sin^2\theta . \frac{\Delta Y}{Y}. \tag{6}$$

It is to be observed, that we are not necessarily restricted to the same course in obtaining the absolute values of the elements at a given epoch, and in determining the changes (periodic, secular, or irregular) which they undergo from one epoch to another. As regards *absolute* measures, the observed elements are the declination, the inclination, and the horizontal component of the intensity, the means of determining the last of which in absolute measure have been supplied by GAUSS. The total intensity is then inferred from the results of observation by means of (1). In the researches connected with the *variations* of the earth's magnetic force, which constitute the main business of an observatory, the observed elements, at Dublin, have been hitherto the declination and the two components of the intensity; and, accordingly, the variations of the total intensity and inclination are inferred by the formulæ (5) and (6). For the vertical component of the intensity, however, I have lately substituted the inclination, as the third of the observed elements; and, in this case, the variations of the total intensity are to be deduced from those of its horizontal component and the inclination, by the formula (3).

The instruments employed in the absolute measures are, the *declinometer*, the ordinary *inclinometer*, and the *unifilar magnetometer*(*a*). Those used in observing the variations are the declinometer (which answers both purposes), the *bifilar magnetometer*, and the *balance magnetometer;* for the last named of which instruments, I have lately substituted the *induction inclinometer*. The declinometer, the unifilar and bifilar magnetometers, and the induction inclinometer, were all made by Mr. GRUBB, of Dublin; the ordinary inclinometer was furnished by M. GAMBEY, of Paris; and the balance magnetometer by the late Mr. ROBINSON, of London. The three instruments used in determining variations are placed in the principal observatory, in the positions indicated in Plate I. fig. 2, in which the supporting pillars are distinguished by the letters A, B, C. The unifilar magnetometer and the inclinometer are placed in the magnetic pavilion.

(*a*) I have adopted the terms "declinometer" and "inclinometer" (rather than the words "ecclinometer" and "cataclinometer"), in order that the thing observed, and the instrument used in observing it, may be designated by similar names; and the composition of the terms seems justifiable on the ground, that their component parts, though belonging originally to different languages, are now both naturalized in the English. The term "magnetometer," introduced by GAUSS, appears to be properly adapted to designate those instruments, which serve to measure the magnetic force, or either of its components.

DECLINOMETER.

The essential part of this instrument is a magnet bar, suspended by fibres of untwisted silk, whose actual position at any instant is observed by means of a telescope at a distance. For this purpose the bar is furnished with a collimator, a lens being attached near one end, and a scale finely divided on glass at its focus, near the other.

In order to measure the *absolute declination*, it is necessary to refer the line of collimation of the observing telescope, first, to the magnetic axis of the bar, and, secondly, to the astronomical meridian. The instruments employed in this double reference are a transit instrument and theodolite, the former of which is fixed near the southern window of the observatory, on the supporting pillars D, D, (fig. 2), and the latter in the meridian of the transit, its centre being at the point E, where this plane is cut by the magnetic axis of the suspended bar. To determine the direction corresponding to the magnetic axis, the bar must be inverted, and the points of the scale coinciding with the vertical wire of the theodolite telescope noted before and after inversal; half the sum of these readings is the point of the scale corresponding to the magnetic axis. This point being ascertained, we may either turn the telescope until it is brought to coincide with the vertical wire; or, which is better, we may compute the deviation of the line of collimation from the magnetic meridian (viz. the difference between the actual reading of the scale and that corresponding to the magnetic axis, converted into angular measure), and allow for it in the result. We have next to refer the line of collimation of the theodolite telescope to the astronomical meridian. For this purpose the transit telescope is turned over, and made to serve as a collimator to the other. The theodolite telescope being directed to its middle wire, through the object glass, their lines of collimation are brought to parallelism by the slow movement attached to the former instrument.

Let the angle read off on the limb of the theodolite be called A, and the deviation of the line of collimation in the first part of the observation D, then, the magnet and transit instrument being on opposite sides of the theodolite, the declination is

$$180^\circ - (A + D).$$

If, instead of the *actual* declination at the moment of observation, we desired the *mean* declination of the day or of the month, we have only to employ (instead of the actual reading of the scale) the corresponding mean result. We should thus obtain a new value of D, differing from the former by the amount of the declination change.

The division of the scale corresponding to the magnetic axis of the bar is to be considered as the *zero point*, and must be determined with great exactness. It is obvious that this point will be given by the mean of two readings of the scale, with the magnet in the *erect* and *inverted* positions, provided that care has been taken to eliminate the declination changes which may occur in the interval of the two parts of the observation. The obvious method of effecting this elimination is to determine the amount of the declination change, by means of an auxiliary apparatus, and to apply it as a correction to the second result. The same thing may, however, be effected by taking a series of readings of the declinometer alone, with the bar alternately erect and inverted; the time chosen for observation being one in which the declination changes are small and regular, and the readings being made in as rapid succession as possible. By comparing each result with the mean of the preceding and subsequent, and then taking the mean of all these partial means, a very accurate determination may be made.

In observing the *declination changes*, the telescope of the theodolite remains fixed; and the angular change is determined by noting the point of the scale coinciding with the vertical wire. As, however, the magnet is in a state of continued vibration, under the action of the earth's force, it is necessary to institute the observations in such a manner as to be able to infer its mean position from the observed readings. The following modification of one of the methods proposed by Gauss, for the attainment of this end, appears to combine the greatest number of advantages,—namely, to take three readings, at the times

$$t-T, \quad t, \quad t+T;$$

t being the epoch for which the position of the magnet is desired, and T its time of vibration. In fact, it has been shown(*a*) that the equation of motion of a vibrating magnet in a resisting medium is of the form

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(*a*) Proceedings of the Royal Irish Academy, June 14, 1841.

$$u = \left(c \cos \pi \frac{t}{T} + c' \sin \pi \frac{t}{T}\right) e^{-At},$$

in which u is the deviation of the magnet from its mean position at the time t, and A a constant depending on the resistance, which is assumed to be proportional to the velocity. This constant being small in the case under consideration, we may take approximately

$$e^{-At} = 1 - At;$$

whence it will be seen, that if $u_,$ and u' denote the values of u, when t becomes $t - T$ and $t + T$ respectively, we shall have

$$u_, + 2u + u' = 0;$$

so that by combining the three readings according to the preceding formula, the deviation of the magnet from its mean position, arising from the vibratory movement, is completely eliminated. Now, let the value of u contain an additional term, $+ pt$, proportional to the time: or, in other words, let us suppose that there is a *progressive* change of the declination, which may be regarded as *uniform* during the whole interval of observation. It is then manifest that

$$u_, + 2u + u' = 4pt;$$

and accordingly that the quantity

$$\tfrac{1}{4}(u_, + 2u + u')$$

will give the mean place of the magnet corresponding to the epoch t.

The supposition of a uniform change can, however, be regarded as an approximation to the truth, only when the interval of time between the first and last reading is very small, in comparison with the interval between the successive maxima and minima, in the fluctuations of the irregular movements. Hence we may conclude, that it is important, in the first place, to employ three readings in preference to any greater number; and, secondly, that it is desirable that the time of vibration of the magnet itself should be as small as possible, consistently with the accuracy of its indications in other respects.

But before the actual changes of the declination can be deduced from the observed readings, it is necessary to apply a correction depending upon the force of torsion of the suspension thread. For it is manifest that the magnet will deviate from the direction of the earth's horizontal force, towards the plane of detorsion; and this deviation will vary with the angle formed by that direction with that plane. Let u denote the angle which the magnetic axis of the bar forms with the direction of the earth's force; and v the angle which it makes with the plane of detorsion: then the moment of the force of torsion is

$$Hv\,;$$

H denoting the force corresponding to the arc whose length is equal to radius. But this force is resisted by the earth's magnetic force, whose moment is

$$mX \sin u\,;$$

X being the horizontal component of the earth's force, and m the moment of free magnetism of the bar. Now, the angle u being always small, we may take $\sin u = u$, u being expressed in parts of radius; so that the equation of equilibrium is

$$Hv = mXu.$$

But the angle which the direction of the earth's force makes with the plane of detorsion is

$$w = v + u = v\left(1 + \frac{H}{mX}\right);$$

and, the plane of detorsion remaining unchanged, it is obvious that the variations of this angle are those of the actual declination, while the changes of the angle v are the corresponding changes of position of the bar. Hence the true changes are expressed, in terms of those observed, by the formula

$$\Delta w = \Delta v\left(1 + \frac{H}{mX}\right).$$

In order to determine experimentally the value of the ratio $\frac{H}{mX}$, it is only necessary to increase the angle of torsion by some known amount, and to observe the corresponding change of position of the magnet. For, from the formulæ above given, we have

$$w = \left(\frac{mX}{H} + 1\right)u, \qquad w' = \left(\frac{mX}{H} + 1\right)u'\,;$$

and subtracting,

$$w - w' = \left(\frac{mX}{H} + 1\right)(u - u').$$

Whence

$$\frac{mX}{H} = \frac{w - w'}{u - u'} - 1.$$

The angle $u - u'$ is read off by means of the scale attached to the magnet; the angle $w - w'$ by means of a divided circle connected with the suspension apparatus.

Construction. The bar is a rectangular parallelopiped, 15 inches in length, $\frac{7}{8}$ths of an inch in breadth, and $\frac{1}{4}$th of an inch in thickness. In addition to the stirrup by which it is suspended (Plate II. fig. 1), it is furnished with two sliding pieces, one of which contains an achromatic lens, and the other a finely-divided scale of glass (figs. 2 and 3). The scale being adjusted to the focus of the lens, the rays proceeding from each point of it are refracted parallel to one another, and to the line connecting that point with the centre of the lens; the apparatus thus becomes a moving collimator, whose absolute position at any instant, as well as its changes of position from one instant to another, may be read off by a telescope at a distance. The aperture of the lens of this collimator is 1$\frac{1}{4}$ inch, and its focal length 12.5 inches. Each division of the scale is $\frac{1}{383}$th part of an inch; and every twentieth division is numbered, by the aid of an ingenious reducing apparatus, contrived by the artist for the purpose. The arc-value of each division is $43''.20$; and the visual angle under which it is seen in the telescope is so considerable, that it may be readily subdivided into tenths by estimation. Thus the readings are made to four seconds nearly.

To the suspension thread is attached a small cylindrical bar, the ends of which are of smaller diameter, and support the stirrup which carries the magnet. The apertures in the stirrup (fig. 1), by which it hangs on the cylinder, are of the form of inverted Ys, so that the bearing points are invariable. A second pair of apertures, at the other side of the magnet, serves for the purpose of inversal; and care has been taken to render the lines connecting the bearing points of each pair of Ys parallel, so that there may be no difference in the amount of torsion of the thread in the two positions of the stirrup. The two pairs of apertures are at different distances from the magnet, in order that the line of collimation may remain nearly at the same height on inversal, and so it may not be necessary to alter the length of the suspension thread. The stirrup, and the other sliding pieces, are formed of gun metal.

For the purpose of taking out the torsion of the suspension thread, the apparatus is furnished with a *detorsion bar*, which (including appendages) is of the same weight as the magnet (fig. 5). It is a rectangular bar of gun metal, furnished with a stirrup and collimator similar to those of the magnet. A rectangular aperture in the middle of the bar receives a small magnet, 4 inches long, the use of which is to impart a slight directive force to the suspended body, without which the final adjustment of detorsion would be tedious and difficult.

The framework of the instrument consists of two pillars of copper, 35 inches in height, which are firmly screwed to a circular marble base, 21 inches in diameter and 1½ inch thick (Plate III. fig. 1). These pillars are connected by two cross pieces of wood, one at the top, and the other 7 inches from the bottom. In the centre of the top piece is the suspension apparatus, the plan of which is similar to that of the torsion balance of COULOMB (Plate II. figs. 6 and 7); it is provided with a divided circle used in determining the amount of torsion of the thread. A glass tube, between this and the middle of the lower cross piece, incloses the suspension thread; and a glass cap at top covers the suspension apparatus, and completes the inclosure of the instrument.

The box which incloses the magnet, to protect it from the agitation of the air, is of wood; its form is cylindrical, its dimensions being 20 inches in diameter, and 7 inches in depth (Plate III. fig. 1). It has no bottom; and the lid being made in two halves, which are moveable, the whole may be raised, when necessary, for the purpose of manipulation. There are two apertures in the box, opposite to each other. The aperture in front, used for reading, is covered with a circular piece of parallel glass, 2 inches in diameter, attached to a rectangular frame of wood which moves in dovetails; the position of the aperture is thus adjusted to that of the collimator, and the prismatic error of the glass (if any) is corrected by simply reversing the slider in the dovetails. The opposite aperture is for the illumination of the scale, and is also covered with glass, attached to a sliding piece of wood. In order to determine the interior temperature, the instrument is provided with a thermometer, inclosed in a tube of glass, which enters the box through an aperture in the lower cross piece. The tube is closed at top, and open below, that the temperature of the air surrounding the bulb and magnet may be the same.

A copper ring surrounds the magnet, when in use, for the purpose of checking its vibrations. This ring (Plate II. fig. 4), is formed of two rectangular bars, of the same breadth and thickness as the magnet, united at the ends by two circular pieces. Its operation is so effective, that the arc of vibration is reduced to less than one-half of its original value in four oscillations.

The theodolite, used in connexion with the declinometer, was made by Messrs. TROUGHTON and SIMMS. The limb of this instrument, which rests upon a massive tripod supported by adjusting screws, is 12 inches in diameter, and is divided to 10′; the readings are made to 10″, by means of three verniers. The whole limb has a movement independent of the motion of the telescope and verniers, so as to enable the observer to *repeat* upon different parts, and thus eliminate the errors of division. The telescope rests upon Ys, and has a small range of motion in the vertical plane. The aperture of its object-glass is 1⅝ inches; its focal length is 18 inches; and the magnifying power usually employed is 39. The telescope is furnished with a micrometer, of which the fixed wire is a single vertical wire, the moveable wires being formed in a cross; the ultimate divisions of the micrometer correspond to two seconds of arc, very nearly. There is a second telescope below the limb, for the purpose of examining whether the position of the instrument has remained invariable, and of correcting for any change which may have occurred in the course of observation; this telescope has a motion in azimuth, with fast and slow movement, for the purpose of bringing its line of collimation to coincide with a fixed mark.

Adjustment.—The instrument having been placed on its support, the base is levelled, and the whole then permanently fixed. The suspension thread is then formed, and attached at one extremity to the roller of the suspension apparatus, and at the other to the small cylinder which bears the stirrup and magnet. Sixteen fibres of untwisted silk are sufficient to bear double the load without breaking, care being taken to give an equal tension to all.

These preparations being made, the adjustments are the following:

1. The sliders being placed on the magnet, the divided scale is adjusted to the focus of the lens, the position of the sliders being such that their centre of gravity may be near the middle of the bar. The adjustment to focus is readily made, by first adjusting the reading telescope to sidereal focus, and then using it to view the scale through the object lens of the collimator; the scale will be in the focus of the lens, when it is distinctly seen with the telescope.

2. The magnet is connected with the suspension thread by means of the stirrup, and moved in the latter until it assumes the horizontal position. This adjustment may be conveniently tested by means of the image of the magnet, reflected from the surface of water or mercury, the reflected image being parallel to the object when horizontal. The stirrup is then fastened by its screws, and the magnet wound up to the desired height.

3. The magnet is then removed, and the unmagnetic bar (having its collimator similarly adjusted) is attached, without its small magnet. The bar having come to rest, its deviation from the magnetic meridian is *estimated*, and the moveable arm of the torsion circle turned through the same angle in an opposite direction. The plane of detorsion then coincides, approximately, with the magnetic meridian.

4. The magnet is then replaced, and the telescope being directed towards the collimator, the point of the scale coinciding with the vertical wire is noted when the magnet is in the *direct* and *inverted* positions. Half the sum of these readings is the point of the scale corresponding to the magnetic axis; and half their difference (converted into angular measure) is the deviation of the line of collimation of the telescope from the magnetic meridian. The telescope is then moved through this angle in the opposite direction.

5. In order to take out the remaining torsion of the thread, the magnet is again removed, and the unmagnetic bar (with its small magnet attached) substituted. The deviation from the magnetic meridian being then read off on its divided scale, the moveable arm of the torsion circle is turned through a known angle in the opposite direction. The deviation being again read, a simple proportion will give the remaining angle of torsion; and the moveable arm being turned through this angle in the opposite direction, a third reading will serve to verify the adjustment. The plane of detorsion then coincides with the magnetic meridian; and the magnet being replaced, the instrument is ready for use.

INCLINOMETER.

The inclination instrument is made by GAMBEY, and is of the usual form and dimensions adopted by that artist. The vertical circle is a ring of copper, supported at the extremities of its horizontal diameter by two short pillars of the same metal, which are fixed below to a rectangular platform. The circle is 9 French inches in its interior diameter; and it is divided on silver to 10′. Two horizontal bars, attached to the upper extremities of the same pillars, carry the agate pieces on which the axis of the needle rests. These pieces are capable of a vertical movement in a rectangular frame of the same size; and their upper surfaces (which are cylindrical) are thus adjusted to the same horizontal plane by screws bearing upon the lower edges. An interior moveable frame serves to lift the needle off the agates. The platform, and the parts connected with it, have a motion in azimuth upon a vertical axis, which plays in a socket in the supporting stem of the instrument; a horizontal circle attached to the top of the same stem, 7 inches in diameter, and read by a vernier to single minutes, serves to measure the azimuth of the plane of the vertical circle. The axis is rendered vertical by the aid of three adjusting screws; and a delicate level on the platform serves to test the adjustment. A detached case, formed of wood and glass, rests upon the platform, and protects the needle during observation from the agitation of the air. Two moveable arms, attached to a horizontal bar connected with the case, carry the reading lenses.

The apparatus is furnished, as usual, with two needles, whose length is 9 French inches.

The plane of the vertical circle is usually made to coincide with the magnetic meridian; in which case the inclination is given immediately by observation, the instrument being supposed free from all sources of error. For the purpose of testing the axles of the needles, and the limb of the instrument, however, it is desirable to make, occasionally, a series of observations in various azimuths. The relation between the magnetic inclination, and the angle which the needle makes with the horizon under these circumstances, is easily deduced; the following is the investigation, as given by M. BIOT(a).

(a) *Traitè de Physique*, tom. iii. p. 24.

The earth's magnetic force, R, being resolved into two, one vertical and the other horizontal, these components are

$$Y = R \sin \theta, \qquad X = R \cos \theta;$$

θ denoting the inclination. If the needle does not move in the plane of the magnetic meridian, the latter force will be, in part, counteracted by the resistance of the supports. To estimate its effect upon the needle, therefore, it must be resolved into two,—one in the intersection of the plane in which the needle moves with the horizontal plane, and the other perpendicular to that line. These components are

$$X \cos a, \qquad X \sin a;$$

in which a denotes the azimuth of the plane in which the needle moves, measured from the magnetic meridian. The latter of these forces being destroyed by the reaction of the supports, the efficient forces are

$$Y = R \sin \theta, \qquad X' = R \cos \theta \cos a.$$

But, if η denote the angle which the resultant of these forces makes with the horizon, we have

$$\tan \eta = \frac{Y}{X'};$$

and, substituting for X' and Y their values,

$$\tan \theta = \tan \eta \cos a.$$

The azimuth, a, is easily determined. For, it follows from this formula, that when $a = 90°$, (that is, when the plane in which the needle moves is perpendicular to the magnetic meridian), $\eta = 90°$, and the needle is vertical. The vertical position of the needle, therefore, serves to determine when the plane of the instrument is perpendicular to the meridian, and the angle between this plane and the plane of observation ($90° - a$) is obtained by the help of the azimuth circle.

We may also infer the magnetic inclination from the observed inclinations of the needle in any two planes at right angles to one another, without the knowledge of the angle a. For, if η and η' denote the observed inclinations, in the planes whose azimuths are a and $90° - a$, we have

$$\cot\eta = \cot\theta\cos a, \quad \cot\eta' = \cot\theta\sin a\,;$$

whence, squaring and adding,

$$\cot^2\theta = \cot^2\eta + \cot^2\eta'.$$

A series of the values of θ being thus obtained from observations in many different azimuths, the difference between the mean, and the result obtained at the same time in the magnetic meridian, furnishes a correction for the errors of axle and limb to be applied to all future observations in the meridian.

UNIFILAR MAGNETOMETER.

The horizontal component of the earth's magnetic force is usually measured, by observing the time of vibration of a suspended magnet under its action. The measure thus obtained, however, is only *relative*. From the observed time of vibration, and the moment of inertia of the bar, we deduce, in fact, the product of the horizontal component of the earth's force into the moment of free magnetism of the suspended bar; and an observation of a different kind is necessary, in order to separate the factors thus combined, and to infer the component of the earth's force in *absolute* measure. If, however, this magnet be made to act upon a second, freely suspended, we may deduce from the observed effects the ratio of its magnetic moment to the component of the earth's force; and the product and ratio being thus known, the two factors are absolutely determined.

Two methods have been proposed for this second observation, one by Poisson, and the other by Gauss. The former of these consisted in observing the time of vibration of the second bar, under the combined action of the first and of the earth, the two magnets having their axes in the same right line. In the second method, which, on the authority of Gauss, has superseded the first, the second magnet is deflected by the first, the acting magnet being placed with its axis in the right line passing through the centre of the suspended magnet, and perpendicular to the magnetic meridian. Theory shows that the force exerted by the magnet is, under these circumstances, directed in the same right line, and is expressed by a series ascending according to the negative odd powers of the distance, commencing with the third. When the distance is greater than four times the length of the magnet, however, the convergence of the series is so rapid that all the terms beyond the second may be neglected. In order to determine the coefficients of the first two terms, the observations must be made at two different distances; and, in each position, the deflection produced is to be observed, with the north end of the deflecting magnet towards the east and west, successively. The observations are then to be repeated on the other side of the suspended magnet, and at the same distances, so that each angle of deflection is the mean of four observed results.

As the observations of vibration and deflection must be made successively, it is necessary to apply a correction for the changes of the intensity which may occur in the interval. This correction is at once given by the *bifilar magnetometer*, which should therefore be observed simultaneously with the two parts of the observation of absolute intensity.

It is unnecessary to enter further into the details of this method, as they are fully given in the valuable memoir of GAUSS(a). It should be observed, however, that the number thus obtained for the force of the earth's magnetism expresses the ratio which that force bears to the *unit of force*,—the unit of force being that which, acting on the unit of *mass*, through the unit of *time*, generates in it the unit of *velocity*. These units are entirely arbitrary. For the unit of mass, we may take a *grain;* for the unit of time a *second;* and, if a *foot* be taken as the unit of space, the unit of velocity will be that of one foot per second. Again, as the magnetic force operates effectively only on the free or uncombined elements of the magnetic fluid, we are to understand by the earth's magnetic force, its action on the elementary unit of free magnetism; and we must take for that unit the quantity of free magnetism, which acting on another equal quantity at the unit of distance, exerts an effect equal to the unit of force already defined.

Construction.—The instrument employed in these measurements is similar to the declinometer in its construction, differing from it only in its dimensions, and in a few other particulars suggested by experience. The magnet bars are 12 inches in length; their breadth is $\frac{7}{8}$ths of an inch, and thickness $\frac{1}{4}$th of an inch, as before. Great care has been taken to give them a perfect form, so that the moment of inertia may be inferred from the dimensions and weight of the bars, without experiment. The object lens of the collimator has an aperture of 1 inch, and focal length of 10 inches. The length of the pillars which form the frame-work of the apparatus is 24 inches, and the interval of their axes 12 inches. The box is rectangular, and is composed of two parts, which fit together by a rabbitted joint; its length is 16 inches, its breadth 8 inches, and depth 6 inches. The piece containing the front aperture is made to slide horizontally (instead of vertically), the stirrup of the magnet being so contrived that the line of collimation is exactly at the same height on inversal.

(a) *Intensitas vis magneticæ terrestris ad mensuram absolutam revocata.*

This instrument is placed in the magnetic pavilion, and is fixed upon a granite pillar which is insulated from the floor. The reading telescope is fixed upon another pillar, at the distance of 6 feet (from centre to centre) in the magnetic meridian. The focal length of the telescope is 13 inches; the aperture of its object-glass is 1.6 inches; and its magnifying power is 27. It is furnished with an additional object lens, fitted in a cap which slides over the other, so as to adapt it to the distance of the end of the magnet, which is observed in experiments of vibration.

For the purpose of placing the deflecting magnet correctly, and of measuring its distance with accuracy, the apparatus is provided with two measuring rods, 6 feet 5 inches in length, which rest at one end on the marble slab of the instrument, and at the other on brackets firmly fixed to the wall, in the line passing through the centre of the instrument and perpendicular to the magnetic meridian. These rods are formed of two pieces of mahogany placed transversely, so as to guard against flexure. Their inner ends are brought into contact with the pillars of the instrument, the plane of which is perpendicular to the magnetic meridian; and the central lines of the rods themselves are brought into the same plane, by the help of a magnetic needle which is placed on a pivot fixed in the central line of the rod, and is made to coincide with a line traced on brass at right angles to it. A rectangular groove, of the same width and depth as the magnet, is cut in each rod; and the height is so arranged that the deflecting magnet, when placed in these grooves, shall be in the same horizontal plane with the suspended magnet. For the purpose of measuring the distance of the deflecting magnet, pieces of brass are inserted at intervals of six inches, on the surface of each of which a fine line is traced at right angles to the groove. A similar line being traced on the magnet, exactly at its centre, the two lines may be at once brought to coincidence. The distances of these lines, from a similar line near the inner end of the rod, are to be verified from time to time; and for this purpose the observatory is furnished with a standard yard, and beam compass, made by TROUGHTON and SIMMS. Finally, the interval of the zero lines, when the ends of the two rods are in contact with the pillars, is measured by a standard scale.

BIFILAR MAGNETOMETER.

The instrument employed in measuring the *variations* of the horizontal component of the earth's magnetic force is similar, in principle, to the "*bifilar magnetometer*" of GAUSS. It is a magnet bar, suspended by two equidistant wires (or, more accurately, by two portions of the same wire, the distance of whose bearing points is the same above and below), and maintained, by the rotation of the upper extremities of the wire round their middle point, in a position at right angles to the magnetic meridian(*a*).

It is manifest from the nature of this suspension, that the weight of the suspended body tends to bring it into the position in which the two portions of the wire are in the same plane throughout. The earth's magnetic force, on the

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(*a*) I think it right to state, in this place, that both the principle of this instrument, and the means adopted by GAUSS in applying it, were previously known in this country. Professor CHRISTIE made use of a magnet, deflected into a position transverse to the magnetic meridian, in observing the diurnal variations of the horizontal intensity, the empirical laws of which he thus determined approximately. The deflecting force, in this case, was the attraction and repulsion of a pair of fixed magnets, placed in the magnetic meridian passing through the centre of the moveable magnet, one to the north and the other to the south; and Professor CHRISTIE has pointed out the use which may be made of the balance of torsion for the same purpose. The bifilar suspension itself originated with Mr. SNOW HARRIS, who substituted it for the balance of torsion in his researches on the laws of electrical forces. GAUSS applied it, nearly in the manner proposed by Professor CHRISTIE with respect to the balance of torsion, although probably without any knowledge of what had been done by the gentlemen above mentioned. To any one acquainted with the memoirs of Professor CHRISTIE and Mr. SNOW HARRIS, this application would naturally suggest itself; and I therefore claim no merit in stating, that I mentioned to the latter gentleman, in the autumn of 1837, my intention of making it; and that I described the method itself in the spring of the following year (*Proceedings of Royal Irish Academy*, April 23, 1838), before the memoir of GAUSS appeared in this country.

other hand, draws the bar into the magnetic meridian; and it will consequently rest in the position in which the moments of these opposing forces are equal. In order to determine the equation of equilibrium, we must, in the first place, ascertain the law of the directive force arising from the suspension.

Let *mn*, *m′n′* (Plate III. fig. 6), represent the two strings in their natural position, in which the lines joining the bearing points above and below, *mm′*, *nn′*, are parallel. In these circumstances the system is in equilibrium, each string bearing half the weight. Now, let one of these lines, *nn′* (it is indifferent which), be twisted into the position *pp′*. The connecting strings, *mp*, *m′p′*, are now no longer vertical; the weight is consequently raised, and its tendency to descend will engender a directive force, urging the system back to its original position. The weight being applied at *o*, the middle point of the line *pp′*, it may be resolved into two equal parts applied at *p* and *p′*; namely, the force $\frac{1}{2}W$ applied at the point *p*, in the vertical line *ps*; and a similar and equal force at *p′*. Let the former of these forces be resolved into two, one in the direction of the string itself, *mpr*, and the other horizontal. The plane of these components is the plane *rps*,—or, *mn* being parallel to *ps*, it is the plane *pmn*, passing through the two positions of the string; hence, the horizontal force acts in the line *pq* perpendicular to *mn*. Accordingly, if we denote the angle *pmn*, formed by the two positions of the string, by *i*, this force is

$$\tfrac{1}{2}W \tan i;$$

the other component is destroyed by the reaction of the fixed point, *m*. Let the point *q* be connected with *o*, the middle point of the line *pp′*. Then, as the latter point rises in a vertical line, it is obvious that *oq* is parallel to *nn′*, the position of quiescence, so that the angle *qop* is the deviation from the plane of detorsion, or the angle formed by the lines joining the bearing points above and below; and, *oq* and *op* being equal, *pq* is the chord of this angle, to the radius *op*. Let this angle be denoted by *v*. The horizontal force acting in the line *pq* must be resolved into two, one in the direction of the radius *po*, which is balanced by the similar component of the other half of the weight; and the other in that of the tangent *pt* (fig. 7), which is the part of the force urging the system back to its position of equilibrium. The latter component is equal to horizontal force × cos *qpt*, or to

$$\tfrac{1}{2}W \tan i \cos \tfrac{1}{2}v.$$

But, when the length of the string is considerable, as compared with the interval pp', the angle i is always small, and we may substitute its sine for its tangent. Now, $\sin i = \frac{pq}{pm} = \frac{2po \times \sin \frac{1}{2} v}{pm}$; wherefore, if we denote half the interval of the threads, po, by a, and their length, pm, by l, we have approximately

$$\tan i = \frac{2a \sin \frac{1}{2} v}{l}.$$

Accordingly, substituting this value in the expression given above, the effective force applied at the point p is

$$\tfrac{1}{2} W \frac{a}{l} \sin v;$$

and its moment to turn the line pp' round the point o, is $\frac{1}{2} W \frac{a^2}{l} \sin v$. A similar and equal force being applied at the point p', the total directive force is

$$W \frac{a^2}{l} \sin v.$$

The directive force arising from the resolved part of the weight being found, it is easy to form the equation of equilibrium. For this force is opposed by that which the horizontal component of the earth's magnetic force exerts upon the bar, i. e. by the force mX, in which X denotes the horizontal component, and m the moment of free magnetism of the bar. And the moment of this force to turn the bar is

$$mX \sin u,$$

u being the angle which the magnetic axis of the bar makes with the meridian. The equation of equilibrium, therefore, is

$$mX \sin u = W \frac{a^2}{l} \sin v;$$

and when the bar is brought into the position perpendicular to the magnetic meridian, in which position the earth's force acts with the greatest mechanical advantage, we have

$$mX = W \frac{a^2}{l} \sin v.$$

As all the quantities involved in the second member of this equation may be known by direct measurement, we may deduce in this way the value of the product of the earth's magnetic force into the moment of free magnetism of the bar, which is usually obtained by experiments of vibration, and thus employ the instrument in measures of *absolute* intensity. The experimental difficulty in this method consists in the determination of the quantity a, which should be known to a very small fractional part of its actual value. This difficulty has been in a great degree overcome by the measuring apparatus connected with the suspension, which serves to determine the interval of the wires, at their upper extremity, to the $\frac{1}{2000}$th of an inch; and the numbers given in the table, (page 33), for the lower interval, may be relied on to the same degree of accuracy. For many reasons, however, this method is practically inferior to the ordinary process.

The chief use of the instrument is, to observe the *variations* of the intensity. The magnet, it has been shown, is acted on by two forces, and rests in the position in which their moments are equal. But one of these forces being variable, the position of equilibrium must vary likewise; and the variations of angle are necessarily connected with the variations of force. Making, for abridgment,

$$mX = F, \quad W\frac{a^2}{l} = G,$$

the equation of equilibrium is

$$F = G \sin v\,;$$

and differentiating with respect to F and v, and dividing by the equation itself, we have

$$\frac{\Delta F}{F} = \cot v \, \Delta v,$$

the angle Δv being expressed in parts of radius. This angle is observed by means of a collimator and fixed telescope, precisely in the same manner as the changes of declination. Let n denote the number of divisions of the scale of the collimator, which is equal to the difference of the readings at any two epochs; then the corresponding variation of the angle is

$$\Delta v = na\,;$$

a denoting the arc-value (in parts of radius) corresponding to a single division, and the numbers being supposed to increase as the angle v increases. Hence the changes of force are given, in parts of the whole force, by the formula

$$\frac{\Delta F}{F} = n\,a \cot v.$$

The quantity F, in the preceding formula, is the product of the earth's magnetic force into the moment of free magnetism of the bar; and, as the latter quantity varies with the temperature, it is necessary to apply a correction, before we can infer the true changes of the force. This correction is easily deduced. Since $F = Xm$, there is

$$\frac{\Delta F}{F} = \frac{\Delta X}{X} + \frac{\Delta m}{m}.$$

Let t denote the actual temperature, in degrees of Fahrenheit; t_0 the standard temperature; and q the relative change of the magnetic moment corresponding to one degree; then

$$-\frac{\Delta m}{m} = q\,(t - t_0).$$

And, substituting in the preceding formula,

$$\frac{\Delta X}{X} = kn + q\,(t - t_0),$$

making, for abridgment,

$$k = a \cot v.$$

Construction.—The magnet bar is of the same dimensions as that of the declinometer. The collimator, by which its changes of position are observed, is inclosed in a light tube attached to the stirrup (Plate III. fig. 2), and has a motion in azimuth; this arrangement enables the observer to choose the place of the reading telescope according to convenience, and also facilitates the adjustment. The aperture of the lens is $\frac{8}{10}$ths of an inch, and its focal length 8 inches, nearly. The divisions of the scale are the $\frac{1}{383}$th of an inch; and the arc-value corresponding to each division is 1′.0776.

The suspending wire is of gold, and its diameter the $\frac{8}{1000}$th of an inch. It passes round a small grooved wheel (fig. 4), on the axis of which the stirrup (fig. 3) rests by inverted Ys; and the instrument is furnished with a series of such wheels, whose diameters increase in arithmetical progression (the common difference being about $\frac{1}{20}$th of an inch), for the purpose of varying the interval of the wires. The intervals of the axes of the wires corresponding to each wheel, were measured by a microscope, moved by a micrometer screw, the cross in the focus of the microscope being made to coincide successively with the corresponding sides of the two wires. In order to avoid the errors arising from flexure, the measurements were made with a cotton thread waxed; and these were reduced to those corresponding to the wire, by a preliminary very exact measurement with both, used on the same wheel, the mean of several results being taken, in which the wire was strained outwards and inwards alternately. The following Table gives the corrected intervals, in decimals of an inch, corresponding to each wheel:

I	II	III	IV	V	VI	VII	VIII
·2549	·3075	·3552	.4068	.4620	.5078	·5571	.6110

The interval of the wires is altered at their upper extremity, by means of two screws (one right-handed and the other left-handed) cut in the same cylinder; the wires being lodged in the intervals of the threads, and their distance regulated by a micrometer head (fig 5). The interval of the threads of this screw is $\frac{2}{77}$ths of an inch. The micrometer head is divided into 100 parts; and, as one revolution of the head corresponds to two threads of the screw, a single division is equivalent to .0005194, or the $\frac{1}{2000}$th of an inch nearly. The micrometer head has been carefully adjusted by the artist, so that the index is at zero, when the interval of the wires is exactly half an inch.

The larger parts of this apparatus,—the box, the framework, and the support,—are similar to those of the declinometer (fig. 1). It is also furnished, like that instrument, with a thermometer, the bulb of which is within the box, for the purpose of ascertaining the interior temperature; and a copper ring used in checking the vibrations. A brass weight, which is occasionally attached below the collimator, supplies the place of a detorsion bar.

The reading telescope is screwed to a bracket, which is firmly attached to the wall in the position designated by the letter F in Plate I., and at the distance of 8 feet from the centre of the magnet. Its focal length is 15½ inches; the aperture of its object-glass is 1.6 inches; and the magnifying power usually employed is 27. The telescope has a small range of motion in the vertical plane by an adjusting screw.

Adjustments.—The instrument being placed on its support, the base is levelled, and the whole apparatus fixed. The wire is then passed under one of the grooved wheels, and its ends attached above to the suspension roller, care being taken to guard against torsion. The collimator (its scale being previously adjusted to focus) is to be screwed on to the stirrup, and the latter attached to the axis of the grooved wheel by means of its Ys. The weight is then attached, and the wires wound upon the roller, until the collimator is at the desired height. Finally, the interval of these wires is equalized at top and bottom by means of the micrometer connected with the suspension apparatus.

These preparations being made, the adjustments are the following:

1. The magnet being introduced into the stirrup, and the weight removed, we are to determine experimentally the angle through which it is necessary to turn the moveable arm of the torsion circle, in order to deflect the magnet from the magnetic meridian to a position at right angles to it, the two positions being merely estimated. The cosine of this angle is, approximately, the ratio of the magnetic force to the torsion force, or the value of the fraction $\frac{F}{G}$. The nearer this ratio is to unity, the more sensible will be the instrument. If, on making the experiment, the ratio should be found to fall below, or to exceed the proper limits, the torsion force must be altered by employing a different wheel, and making the corresponding alteration in the interval of the upper extremities of the wires.

2. The magnetic axis being brought, approximately, into the magnetic meridian, by turning the moveable arm of the torsion circle, the collimator is turned, by its independent motion, until some point about the middle of the scale coincides with the vertical wire of the fixed telescope. This point of the scale is noted in the usual manner.

3. The magnet being then removed, and the brass weight attached, the new

point of the scale which coincides with the wire of the telescope is noted. Then, if the magnet had been placed, in the previous experiment, in its *direct* position (i. e. north end to north) the error of the plane of detorsion is

$$\left(\frac{G}{F}+1\right)v,$$

v being the difference of the two readings, converted into angular measure(*a*). If, on the other hand, the magnet had been *reversed* (i. e. north end to south), the error is

$$\left(\frac{G}{F}-1\right)v.$$

The moveable arm of the torsion circle is then turned through this angle, in the opposite direction; and the magnetic axis is in the magnetic meridian. The difference of the two readings, corresponding to a given error, being much greater in the reversed than in the direct position of the magnet, it follows that the former affords a much more delicate method of making the desired adjustment.

4. The brass weight remaining attached, the moveable arm of the torsion circle is turned through 90°. The collimator is then turned back, until some point about the middle of the scale coincides with the vertical wire of the fixed telescope; and the reading is noted.

5. The brass weight is now removed, and the magnet replaced. The magnetic force of the earth will bring it back towards the magnetic meridian, and the scale will be thrown out of the field of the telescope. Then the moveable arm of the torsion circle is turned, in the same direction as before, until the point of the scale last noted is brought to coincide again with the wire of the telescope; the magnetic axis is then in the plane perpendicular to the magnetic meridian, and the adjustment is complete.

(*a*) In fact, the general equation of equilibrium being $F \sin u = G \sin v$, it becomes in the case under consideration, in which u and v are both small,

$$Fu = Gv.$$

But the deviation of the plane of detorsion from the magnetic meridian is $u + v$, the magnet having been placed in the direct position; while it is $u - v$, when the position of the magnet had been reversed. Wherefore, substituting for u its value deduced from the preceding formula, we obtain the expressions given above.

BALANCE MAGNETOMETER.

The principle of the instrument, which has been devised for the observation of the changes of the vertical component of the magnetic force, consists in balancing that component by a fixed weight, and observing the changes of the position of equilibrium under the action of the changing force. It is accordingly a magnetic needle resting on agate planes, by knife-edges, and brought to the horizontal position by an adjustable weight.

Let q denote the quantity of free magnetism in a given elementary portion of the needle, and r the distance of that element from the centre of motion. Then, the earth's magnetic force being denoted, as before, by R, the force which it exerts upon the element is Rq; and the statical moment of that force is Rqr. Hence, the sum of all these elementary actions, or the moment of the whole force exerted by the earth on the needle, is

$$R \int qrdr;$$

the integral being taken between the limits $r = \pm$ half the length of the needle. This integral is constant as long as the magnetism of the needle remains unchanged; let it be denoted, for abridgment, by m.

When the plane in which the needle moves is not coincident with the magnetic meridian, a portion of the force Rm is destroyed by the resistance of the supports. It has been already shown (page 24) that the efficient components of the force R are

$$Y = R \sin \theta, \qquad X' = R \cos \theta \cos a;$$

the former being vertical, and the latter in the intersection of the plane in which the needle moves with the horizontal plane. Let η denote the angle which the needle actually makes with the horizon; then the moment of these forces to turn it is

$$m\,(Y \cos \eta - X' \sin \eta).$$

But, if the centre of gravity and centre of motion be not coincident, the force of gravity also exerts an effect to turn the needle, whose moment is

$$Wg \cos (\eta - \epsilon),$$

W being the weight of the needle, g the distance of the centre of gravity from the centre of motion, and ϵ the angle which the connecting line makes with the magnetic axis. Hence, the equation of equilibrium is

$$m\,(Y\cos\eta - X'\sin\eta) = Wg\cos(\eta - \epsilon).$$

When the weight is so adjusted as to bring the needle with the horizontal position, $\eta = 0$, and we have

$$mY = Wg\cos\epsilon.$$

Now, let the forces Y and X' be supposed to alter by a small amount, then the angle η will vary likewise; and the relation which exists among these variations will be obtained by differentiating the equation of equilibrium above given. We have, therefore,

$$m\,(\Delta Y\cos\eta - \Delta X'\sin\eta) - m\,(Y\sin\eta + X'\cos\eta)\,\Delta\eta$$
$$= -Wg\sin(\eta - \epsilon)\,\Delta\eta.$$

And, when $\eta = 0$,

$$m\,\Delta Y = (mX' + Wg\sin\epsilon)\,\Delta\eta\,;$$

or, substituting for X' and Wg, their values $Y\cot\theta\cos a$ and $\frac{mY}{\cos\epsilon}$ derived from the preceding equations,

$$\frac{\Delta Y}{Y} = (\cos a\cot\theta + \tan\epsilon)\,\Delta\eta.$$

Accordingly, the changes of the vertical force are inferred from the observed changes of angle, $\Delta\eta$, by multiplying by a constant coefficient.

If the plane in which the needle moves is coincident with the magnetic meridian, $a = 0$; and denoting, for abridgment, the coefficient in the preceding formula by A, we have

$$A = \cot\theta + \tan\epsilon.$$

In this case, therefore, the greatest value of $\Delta\eta$, corresponding to a given variation of the force, is when $\tan\epsilon = -\cot\theta$, or $\epsilon + \theta = 90°$. When, on the other hand, this plane is perpendicular to the magnetic meridian, there is simply

$$A = \tan\epsilon\,;$$

the horizontal force producing, in this case, no effect; and the greatest value of $\Delta\eta$, corresponding to a given change of force, is when $\epsilon = 0$, or the centre of gravity in the magnetic axis.

The instrument is so contrived as to measure the changes of angle, $\Delta\eta$, with great exactness; but before we can infer the changes of force, by the preceding formulæ, it is necessary to know the angles a, θ, and ϵ, which enter the coefficient. Of these the two former are easily determined by observation. But, in order to determine the latter,—i. e. the angle which the line joining the centre of gravity and centre of motion makes with the axis,—it would be necessary to observe the position of the needle when inverted on its supports; and for this purpose the axle should have the form of the section of a cylinder, the axis of which coincided exactly with the knife-edge. The accuracy required in the construction of this part, however, is such, as to present almost insuperable difficulties in practice; and it fortunately happens that the determination in question may be superseded, and the coefficient obtained by a much simpler process.

To understand this, let us suppose that the needle is disturbed from its position of equilibrium, through any small angle η; it will then, when left to itself, return to that position with an accelerated velocity, and vibrate on either side according to the law of the pendulum. The time of this vibration will serve to determine the coefficient in question. In fact, it will be easily seen that the moment of the moving force, brought into play by this disturbance, is opposite and exactly equal to the change of the vertical component of the magnetic force, which would suffice to bring the needle into that position(*a*); so that this moment is

(*a*) The truth of this proposition may be fully demonstrated by reference to the general expression for the moment. F being the moment of the acting forces corresponding to the inclination η, and $F + \partial F$ that corresponding to the inclination $\eta + \partial\eta$,—when η is the position of equilibrium, $F = 0$, and consequently the moment corresponding to the disturbed position $\eta + \partial\eta$, is obtained by differentiating the general expression for F with respect to the variable η. But

$$F = m(Y\cos\eta - X'\sin\eta) - Wg\cos(\eta - \epsilon);$$

whence

$$\partial F = -[m(Y\sin\eta + X'\cos\eta) - Wg\sin(\eta - \epsilon)]\,\partial\eta.$$

And this, when $\eta = 0$, becomes

$$\partial F = -(mX' + Wg\sin\epsilon)\,\partial\eta.$$

$$-m\Delta Y = -AmY\eta;$$

η being small. Hence the equation of motion is

$$\frac{d^2\eta}{dt^2}+\frac{AmY}{K}\eta = 0,$$

K being the moment of inertia. The integral of this equation is

$$\left(\frac{d\eta}{dt}\right)^2 = \frac{AmY}{K}(\eta'^2-\eta^2),$$

η' being the limit of the arc of vibration; from which, again integrating, we find

$$T = \pi\sqrt{\frac{K}{AmY}},$$

T being the time of an entire vibration.

In order to determine the moment of inertia, K, let the needle be vibrated in the horizontal plane about the same axis. The acting force being in this case the horizontal component of the magnetic force, X, we have

$$T' = \pi\sqrt{\frac{K}{mX}},$$

T' being the time of vibration in the horizontal plane. Squaring the preceding equations, and dividing, we find

$$\frac{T'^2}{T^2} = \frac{AY}{X} = A\tan\theta;$$

whence

$$A = \cot\theta\,\frac{T'^2}{T^2}.$$

Accordingly the coefficient required is known, when we know the mean inclination, and the times of vibration of the needle in the vertical and horizontal planes.

The changes of angle, $\Delta\eta$, are observed with micrometer microscopes. Let n denote the number of divisions of the micrometer head, corresponding to this angle; and a the angle (in parts of radius) corresponding to a single division; then

$$\Delta\eta = na.$$

Hence if we make, for abridgment,

$$k = Aa = a \cot \theta \frac{T'^2}{T^2},$$

the changes of force will be expressed, as in the case of the other component, by the formula

$$\frac{\Delta Y}{Y} = kn.$$

We have here supposed that the magnetic moment of the needle is unaltered during the observations. As, however, this quantity undergoes a change with every variation of temperature, a correction is required before we can infer the actual changes of the earth's force. This correction is precisely similar to that applied in the observations of the horizontal component of the magnetic force; and the complete formula of reduction is, as before,

$$\frac{\Delta Y}{Y} = kn + q\,(t - t_0)\,;$$

where t denotes the temperature in degrees of Fahrenheit, at the time of observation, t_0 the standard temperature, and q the relative change of the magnetic moment of the needle corresponding to one degree.

Construction.—The magnetic needle is 12 inches in length. (Plate IV. figs. 3 and 4.) It has a cross of wires at each end, attached by means of a small ring of copper; the interval of the crosses is 13 inches. The axle of the needle is formed into a knife-edge, the edge of which passes as nearly as possible through the centre of gravity of the unloaded instrument. The weights by which the adjustments are effected are small brass screws moving in fixed nuts, one on each arm, the axis of one of the screws being parallel to the magnetic axis of the needle, and that of the other perpendicular to it.

The larger parts of the apparatus are shown in Plate IV. figs. 1 and 2, the former figure being an elevation, and the latter a plan of the instrument. The agate planes upon which the needle rests are attached to a solid support of copper, which is firmly fixed to a massive marble base. The needle is raised from the agate planes, by means of a horizontal rectangular frame, attached to the top of two upright pieces connected with the supporting stem. These are raised or lowered by an excentric piece passing beneath them, the excentric being made to revolve by a key. The whole is covered with an oblong box of mahogany, in one side of which are two small glazed apertures, for the purpose of reading; the opposite side is covered with plate-glass. A thermometer, within the box, shows the temperature of the interior air; and a spirit level, attached to the marble base, serves to indicate any change of level which may occur in the instrument.

The position of the needle at any instant is observed by means of two micrometer microscopes, one opposite each end. These microscopes are supported on short pillars of copper, attached to the base of the instrument (figs. 1 and 2). They are so adjusted that one complete revolution of the micrometer screw corresponds to about 5 minutes of arc; and, the micrometer head being divided into 50 parts, the arc corresponding to a single division is consequently about 0′.1. The range of these micrometers is three degrees.

In addition to these parts, the apparatus is provided with a brass bar of the same length as the magnet, furnished, like it, with cross wires at the extremities and knife-edge bearings, for the purpose of determining the zero points of the micrometers (fig. 5); a brass rod, of the same length as the magnet, the ends of which are graduated to 10′, used in ascertaining the value of the micrometer divisions; and a horizontal needle, employed in determining the azimuth of the vertical plane in which the needle moves.

Adjustments.—The following are the adjustments of this instrument:

1. The instrument being placed on its support, the azimuth of the plane in which the needle moves is adjusted in the following manner:—That plane is made to coincide, in the first instance, with the magnetic meridian, by means of the horizontal needle which moves upon a pivot fixed to the top of the brass rod. A small theodolite is then placed on the base; and its telescope brought to bear on a distant mark. The telescope is then moved through a horizontal angle equal to the intended azimuth of the instrument, but in an opposite direction; and the base of the instrument is afterwards turned, without disturbing the theodolite, until the mark is again bisected by the wire of the telescope. The plane is then in the required azimuth. The base is then levelled, and permanently fixed.

2. The fixed wires of the microscopes are adjusted to the same horizontal

line by means of the brass needle. This needle being placed upon the agate planes, by its knife-edges, and allowed to come to rest, it is manifest that the line joining the cross-wires will be horizontal, provided it be perpendicular to the line joining the centre of gravity and the axle. To effect this latter adjustment, the needle (a great part of whose weight is disposed below the knife-edge) is furnished with a small moveable weight. The test of the adjustment is similar to that of the corresponding adjustment of the ordinary balance. The moveable wire of one of the microscopes being brought to bisect the cross, if the adjustment is complete, it will bisect the cross at the other extremity upon reversal; if not, the position of the needle will indicate in what manner the weight is to be moved. A horizontal line being thus obtained, the fixed wires of the microscopes are adjusted to it, by moving the capstan-headed screws with which they are connected.

3. The last adjustment is that of the magnetic needle to the horizontal position. To effect this, the needle is furnished with two moveable weights, one on each arm. These weights (it has been already stated) are screws moving in fixed nuts, one in a direction parallel to the magnetic axis of the needle, and the other in a direction at right angles to it; by the movement of the former the needle is brought to the horizontal position, and by that of the latter, the centre of gravity is made to approach the centre of motion, and the sensibility of the instrument thereby increased at pleasure. The time of vibration of the needle upon its knife-edges will serve to measure the sensibility, and thus to test this part of the adjustment

It is manifest that the line joining the two crosses at the ends of the needle will, in general, deviate from the magnetic axis. The amount of this deviation may be ascertained (before the instrument is permanently fixed) by reversing the needle on its supports, the plane of the instrument being perpendicular to the magnetic meridian. Half the difference of the readings, in the two positions, is the deviation sought.

Unexceptionable as the principle of this instrument is in theory, the accuracy of the results has not been commensurate with that of the others. This inferiority is owing to the large influence which the unavoidable errors of workmanship necessarily have on the position of equilibrium of a magnet supported on a fixed axle. It has been shown that the effect of magnetizing a bar, under the most advantageous circumstances of form, and at the part of the globe where the vertical component of the magnetic force is greatest, is the same (as to its position of equilibrium) as if its centre of gravity had been transferred about the $\frac{1}{40}$th of an inch towards the north end; so that the moment of the force, exerted by the vertical component of the earth's magnetism, can never exceed this small quantity multiplied by the weight of the bar. Now, in order to render the results of this instrument comparable to those of the bifilar magnetometer, it should enable us to measure changes of the vertical force, amounting to the $\frac{4}{100{,}000}$dth part of the whole; i. e. we have to measure effects, such as would be produced by shifting the centre of gravity through the *one-millionth of an inch.* It will be easily understood, from this statement, how great must be the effect of a minute disturbance of the relative parts of the instrument, or of inequalities in the bearing points of the axle; and experience has accordingly shown that it is altogether unavailable for the determination of changes of long period.

The same difficulties, and from the same source, have been found to attach to the usual method of observing the magnetic inclination, and its changes, however refined the construction of the instrument. The sources of error seem, in fact, to be inherent in every direct process of determining the third element; and it is only by an *indirect* method that we can hope to evade them. Such is the character of the method which I have now substituted.

INDUCTION INCLINOMETER.

When a soft iron bar, devoid of magnetic polarity, is held in a vertical position, it immediately becomes a temporary magnet under the inducing action of the earth's magnetic force, the lower extremity becoming a north pole, and the upper a south pole. Accordingly, if a freely-suspended horizontal magnet, whose dimensions are small in comparison with those of the bar, be situated near, in a plane passing through one of these poles, it will be deflected from the magnetic meridian. The deflecting force is the induced force of the bar, which may be regarded as proportional to the energy of the inducing cause, i. e. to the vertical component of the earth's force; while the counteracting force is the horizontal component of the same force, acting directly on the magnet itself, to bring it back to the magnetic meridian. Thus the magnet will take up a position of equilibrium, under the action of these opposing forces; and this position will serve to determine the ratio which subsists between them. When the right line connecting the centre of the horizontal magnet, and the acting pole of the bar, is perpendicular to the magnetic meridian, the tangent of the angle of deflection will measure the ratio of the two forces, and will therefore be proportional to the tangent of the magnetic inclination. Accordingly, by observing the changes of position of the horizontal magnet, so circumstanced, we can infer those of the inclination itself(*a*).

But the iron bar may have (and generally will have) a certain portion of *permanent* magnetism, which will concur with the induced magnetism in producing the deflection; and it becomes necessary to institute the observations in such a manner, as to be able to eliminate the effects of this extraneous cause. For this purpose we have only to invert the bar, so that the acting pole, which was uppermost in one part of the observation, shall be lowermost in the other. The induced polarity will, under these circumstances, be opposite in the two cases; and the acting force will in one case be the *sum* of the *induced* and *permanent* forces, and in the other their *difference.*

Let X and Y denote the horizontal and vertical components of the earth's magnetic force, M the intensity of the permanent magnetism in the acting pole of the bar, and m the magnetic moment of the suspended magnet. The intensity of the induced magnetism is, by hypothesis, equal to

$$kY,$$

k being an unknown constant; and when this is of the same name as the permanent magnetism, the intensity of the acting force, at the unit of distance, is

$$kY + M.$$

Accordingly, the moment exerted by this force to turn the suspended magnet is $(kY + M)\, mr \cos u$, u being the angle of deflection, and r a constant depending on the distance: or, making, for abridgment, $kr = p$, $Mr = q$, it is

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(*a*) Professor WEBER has already devised an instrument for the measurement of the absolute inclination, founded on an analogous principle. In this instrument, the deflecting force is the electrical current, generated by the earth's inducing action in a copper ring, which is made to revolve with a known velocity.

$$(pY+q)\, m \cos u.$$

But this deflecting force is resisted by the earth's horizontal force, the moment of which to turn the magnet is

$$Xm \sin u\,;$$

and the magnet will rest when these moments are equal. Hence the equation of equilibrium is

$$pY+q = X \tan u.$$

By the same reasoning it will appear, that when the induced and permanent magnetisms are of contrary names, there is

$$pY-q = X \tan u'\,;$$

in which u' is the new angle of deflection when the bar is inverted. Adding these equations together, and observing that $Y = X \tan \theta$, θ being the inclination, we have

$$2p \tan \theta = \tan u + \tan u'.$$

This equation would furnish at once the inclination sought, provided we knew the value of the constant k. In order to determine it, we have only to place the iron bar *horizontally* in the magnetic meridian, its acting pole remaining in the same place as before, but pointing alternately to the north and south. The inducing force is, in this case, the horizontal component of the earth's magnetic force; and it will be readily seen that the equations of equilibrium are similar to those deduced above, substituting X for Y. If therefore v and v' denote the angles of deflection in these positions, we have

$$2p = \tan v + \tan v'\,;$$

and dividing the equation last found by this,

$$\tan \theta = \frac{\tan u + \tan u'}{\tan v + \tan v'}.$$

Thus, from the deflections produced in these four positions of the bar, we obtain the inclination.

In order to determine the *changes* of the inclination, it is not necessary to observe the deflections in the horizontal position of the bar. Let the equation

$pY + q = X \tan u$ be differentiated, X, Y, and u being all variable, and let the resulting equation be divided by $2p \tan \theta = \tan u + \tan u'$. We thus obtain the following equation, from which p and q are both eliminated:

$$\frac{\Delta Y}{Y} = \frac{2\Delta u}{\cos^2 u\,(\tan u + \tan u')} + \frac{2 \tan u}{\tan u + \tan u'}\,\frac{\Delta X}{X}.$$

But from the relation $Y = X \tan \theta$, we have

$$\frac{\Delta Y}{Y} = \frac{\Delta X}{X} + \frac{\Delta \theta}{\sin \theta \cos \theta};$$

and substituting,

$$\frac{\Delta \theta}{\sin 2\theta} = \frac{\cos u' \Delta u}{\cos u \sin (u + u')} + \tfrac{1}{2}\,\frac{\sin (u - u')}{\sin (u + u')}\,\frac{\Delta X}{X}.$$

The second term of the right-hand member of this equation contains a correction required for the simultaneous changes of the horizontal intensity; but this correction will be generally small, and when the bar has no permanent magnetism, will vanish altogether. In this latter case, in fact, it appears from the equations of equilibrium given above that $u' = u$; so that the preceding formula is reduced to

$$\Delta \theta = \frac{\sin 2\theta}{\sin 2u}\,\Delta u.$$

It must be remembered that the angle u in the preceding formulæ, is the deviation of the suspended magnet from the position which it would assume under the action of the earth alone; and consequently that its changes are the *differences* between the observed changes of position, and the *corresponding changes of declination.* Let a denote the deviation of the suspended magnet, measured from some fixed line, and a' the corresponding angle when the iron bar is removed; then

$$u = a - a', \quad \Delta u = \Delta a - \Delta a'.$$

But $\Delta a = kn$, $\Delta a' = k'n'$; in which n denotes the number of divisions of the scale of the instrument corresponding to the angle Δa, n' the number corresponding to the angle $\Delta a'$, as shown by the declinometer, and k and k' the arc-values of a single division in each instrument. Hence

$$\Delta u = kn - k'n'.$$

Construction.—The magnet is cylindrical; its length is 3 inches, and diameter $\frac{1}{4}$th of an inch (Plate V. fig. 1). A circular mirror, $\frac{3}{4}$ths of an inch in diameter, is attached to the stirrup by which it is suspended, by means of which the varying position of the magnet is observed with a telescope at a distance, after the method of Gauss. The part of the stirrup to which it is attached has the form of a cross; and it is rendered vertical by means of three screws, near the extremities of three of the arms of the cross, the heads of which project and hold it, the mirror being maintained in contact with these heads by springs at the back. The cross itself has a motion round a vertical axis, by means of which the mirror may be adjusted to any desired position of the observing telescope.

The box is octagonal (figs. 2 and 3); the interval between the opposite sides is 4 inches, and that between the top and bottom 2 inches. The top and bottom, and the connecting pillars, are formed of gun-metal; the eight sides are closed by moveable pieces, three of which are of glass, and the rest of ebony. To the top of the box is attached an upright tube of glass (fig. 2), 8 inches in length, which incloses the suspension thread. The suspension apparatus at the top of the tube is of the usual construction; the circular piece to which it is attached has a movement of rotation, and its outer surface is graduated to 5°, for the purpose of determining the effect of torsion of the suspension thread.

The base of the instrument is a circle of gun-metal, 6 inches in diameter, graduated on the edge (figs. 2 and 3). The box is connected with this circle by a short conical stem, forming the axis of a second plate, which revolves upon the fixed one. This moveable plate carries two verniers, by which the angle of rotation may be read off to minutes. Two tubular arms, slightly inclined to one another, are attached to this plate; and their other extremities are connected by a cross-piece, which carries a short scale at the distance of 18 inches from the mirror. This part of the apparatus is employed in determining the total angles of deflection.

The soft iron bar is a cylinder, 12 inches long, and $\frac{3}{4}$ths of an inch in diameter. One of its extremities is inclosed in a hollow cylinder of brass, connected with a horizontal pivot which revolves in a fixed socket. The axis of this pivot being in the line passing through the centre of the suspended magnet, and perpendicular to the magnetic meridian, it is obvious that the bar has a movement of rotation in the plane of the magnetic meridian itself. The distance of the axis of the bar from the centre of the magnet is about five inches; and it is so placed that the induced pole is in the direction of the axis of the pivot, and thus remains fixed during the movement of the bar.

The changes of position of the suspended magnet are observed at a distance by means of a fixed telescope and scale (figs. 4 and 5). The telescope has an aperture of 1⅙ inches, and magnifying power of 22. It has a small range of motion in the vertical plane, by means of an adjusting screw. The scale, whose divisions are reflected by the mirror, is attached above the telescope, at the distance of 36.33 inches from the mirror. The divisions of the scale are engraved on paper; the linear magnitude of 1 division is .02013 of an inch; and the corresponding arc-value is consequently 57″.14.

SUBSIDIARY INSTRUMENTS.

These are a Transit instrument, made by Dollond; a Chronometer, by Arnold and Dent; and a Clock, by Crosthwaite.

The *transit instrument* serves the double purpose of determining the time, and of furnishing a meridian line, in observations of absolute declination. It is fixed close to the southern window, so as to command a view of the equatoreal stars on the meridian, being supported on two stone pillars, five feet in height, and ten inches square (Plate I. fig. 2). A small aperture in the roof, closed with a shutter fitting over a raised edge, allows the telescope a range of 11 degrees on the north side of the zenith (from 48° to 59° of altitude), so as to take in the upper and lower transits of α and δ *Ursæ Minoris*, in adjusting to the meridian. The object-glass of the telescope has an aperture of 2¾ inches, and focal length of 44 inches; the magnifying power employed is 55. The wire-plate in the eye-end is furnished with the usual adjustment for collimation; and the eye-piece is attached to a slider which moves in dove-tails, so as to obtain a perfect view of each of the wires.

The *chronometer* is furnished with a compensation balance of platinum and silver (instead of steel and brass), in order that it may not be acted on by the magnets of the observatory.

The *clock* has been provided with an apparatus, which serves to keep a registry of the actual times of observation. For this purpose it is furnished with a skeleton dial, which is made to revolve once in twenty-four hours; this dial is divided near its edge into 24 principal divisions, and these are subdivided into 12 parts, each of which, therefore, corresponds to 5 minutes. The time is observed by the passing of the divisions before a fixed line, and may be readily noted to single minutes. The observer registers his attendance with a pencil, upon a ring of card, which is fitted to the dial. This ring is similarly divided into 24 parts, and is further divided by concentric circles into 7 annuli, corresponding to the seven days of the week. The aperture through which the observer marks the card, has a height equal to the interval of the successive rings ($\frac{1}{8}$th of an inch); and the piece with which it is connected is so arranged, as to be lifted from one annulus of the card to the succeeding at the end of twenty-four hours. In this manner, a single card contains the registry of an entire week.

METEOROLOGICAL INSTRUMENTS.

Barometers.—The Observatory is provided with two barometers, one made by Newman, and the other by Pistor, of Berlin. They are fixed to the eastern wall of the principal room, in the positions marked by the letters H and K (Plate I). The upper surface of the cill of the northern window was connected with the chain of levels of the ordnance survey, through the kindness of Colonel Colby, and was found to be at a height of 24 feet above low water of ordinary spring tides. The lower surface of the mercury in the barometers is $5\frac{1}{2}$ inches above this cill; and is, therefore, 24.5 feet above low water spring tides.

The English barometer is similar to that in the possession of the Royal Society, which has been fully described by Mr. Baily(*a*). The diameter of the tube is 0.551 of an inch; and the correction for capillary action is therefore + .003. The length of the scale, from the extremity of the agate point to the division marked 30 inches, has been compared with the Astronomical Society's standard, by Mr. Simms, and the error found to be only .0001 of an inch. And,

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(*a*) *Philosophical Transactions*, for 1837.

in the month of June, 1838, previously to its being sent here, the instrument was hung up beside the standard barometer of the Royal Society, and a careful comparison of the two instruments made by Mr. ROBERTON, of which the following Table exhibits the results:

Barom. R. S. Flint-Glass.	Attached Therm.	Barom. Observatory.	Attached Therm.	Difference.
30·080	65·0	30·084	65·6	+ ·004
30·140	65·6	30·142	65·8	+ ·002
30·142	66·2	30·144	66·5	+ ·002
30·196	75·2	30·196	72·2	·000
30·164	69·4	30·166	69·8	+ ·002
30·050	67·6	30·048	68·2	— ·002
29·990	69·3	29·994	69·8	+ ·004
30·010	69·3	30·008	67·8	— ·002
29·982	67·4	29·984	67·6	+ ·002
29·968	65·7	29·968	66·3	·000
30·182	70·2	30·184	69·0	+ ·002
30·100	69·3	30·100	69·8	·000
30·086	71·7	30·086	72·3	·000
30·030	75·7	30·032	73·3	+ ·002
29·766	70·6	29·772	71·6	+ ·006
29·752	70·8	29·762	71·7	+ ·010
29·762	72·6	29·764	71·8	+ ·002
29·844	69·7	29·844	70·2	·000
30·096	77·8	30·094	76·4	— ·002
30·156	70·6	30·156	71·3	·000
30·252	79·6	30·250	76·4	— ·002
30·144	69·6	30·142	70·0	— ·002
30·116	67·0	30·118	67·8	+ ·002
30·226	69·0	30·226	69·6	·000
30·266	70·7	30·264	70·8	— ·002

The differences in the last column of this Table are those of the observed heights, uncorrected for differences of temperature. The two instruments having had nearly the same exposure, it is probable that the actual differences of the temperature of the mercury were less than the errors of the thermometers, so that a correction for the observed differences of temperature would introduce error. The correction for capillarity is also the same in the two instruments, to the third place of decimals inclusive. The agreement of the results is very satisfactory, the mean difference being only + .001.

The German barometer is of the syphon form. The tube is inclosed in a mahogany frame; and is bent in such a manner that the two legs of the syphon are in the same right line. The whole instrument is capable of motion round a vertical axis by means of two pivots attached to the frame at the top and bottom. These pivots move in brass sockets, which are connected with the supporting board by projecting arms; and the upper socket has a motion of adjustment in the horizontal plane, by which the axis on which the instrument turns may be rendered vertical.

The heights of the mercury in the two legs of the tube are observed by microscopes. These are attached to a flat rod of brass, 33 inches long, which is moveable (with a clamp and slow movement) on a piece of the same metal fixed to the mahogany frame. The lower microscope is fixed, near the bottom of the moveable rod. The upper microscope is moveable, with a clamp and slow movement; and the distance of its line of collimation from that of the lower microscope is read on a divided scale attached to the brass rod. This scale is divided on silver to the $\frac{1}{50}$th of an inch; and the readings are made to the $\frac{1}{1000}$dth of an inch, by means of a vernier connected with the part which carries the microscope. In order to adjust the distance of the microscopes, so that the interval of their lines of collimation may correspond with that read on the scale, the lower microscope has a small range of motion by an adjusting screw. The microscopes themselves are compound; and the reading is made by two parallel wires in the focus, the surface of the mercury being made to bisect the interval of the wires, according to the method adopted in Germany in most astronomical instruments. In making the reading, a plate of brass moveable on a hinge is brought opposite to the object-lens of the microscope, on the other side of the tube; by which means the false light is excluded, and the curvilinear surface of the mercurial column is seen brightly illuminated on a dark field.

In order to examine whether the vacuum is perfect at the top of the tube, and to correct for the elasticity of the included air, if it be not so, there is a contrivance for diminishing the space above the mercurial column, and for measuring the amount of the diminution. This measurement is effected by means of a second scale, 3½ inches long, about the middle of the moveable brass rod; the scale being divided in the same manner as the upper one, and read by a vernier attached to the frame. The correction, if any, is inferred from two observed heights of the mercurial column, the ratio of the spaces occupied by the air above being known.

The instrument is furnished with two thermometers, one within the wooden frame, and the other attached to the brass rod which carries the microscopes, the bulb of the latter being in contact with the brass.

Thermometers.—The instruments for the measurement of temperature are, an ordinary thermometer, made by NEWMAN; a maximum and a minimum thermometer for the temperature of the air, made by ADIE of Liverpool; and a maximum and a minimum thermometer for solar and terrestrial radiation, made by NEWMAN. The four latter instruments are self-registering, on RUTHERFORD'S principle.

The first of these instruments has a range of from 5° to 120°. The length of the tube is 14 inches; and the length of a degree on the scale a little greater than the $\frac{1}{10}$th of an inch. It is placed outside the window facing the north; and is covered (except in front of the scale) with a large vessel of thin copper, to guard against radiation. This vessel is polished and varnished externally, and blackened within; and a free current of air is permitted through a number of small apertures (in addition to the front opening) properly arranged at top and bottom. The bulb of the thermometer is five inches distant from the vessel at the nearest part.

The maximum and minimum thermometers for the temperature of the air are of the usual construction. They are 11 inches in length, and the scales are divided on platinum, the length of one degree being $\frac{1}{10}$th of an inch nearly. They are also placed outside the window facing the north, and are guarded by a copper case, similar to that just described.

The maximum thermometer, for the measurement of solar radiation, is a mercurial thermometer with a blackened bulb, registered in the usual manner by a steel index. The minimum thermometer for terrestrial radiation is a spirit thermometer, the bulb of which is placed in the focus of a parabolic mirror, six inches in diameter, which is exposed to the sky; the scale is graduated on the tube itself.

Hygrometers.—The instruments with which the Observatory is furnished, for the measurement of the moisture contained in the air, are the moist-bulb hygrometer, and the hygrometer of DANIELL.

The former of these instruments, which is that regularly employed, is of the same dimensions as the thermometer for the measurement of the air's temperature, and is inclosed within the same protecting case. The two instruments have been carefully compared by a long series of observations made previously to the moistening of the bulb of the hygrometer. The bulb is kept moistened in the usual way, by the capillary action of a piece of silk, which enters a tube containing distilled water through an aperture near the bottom.

From the indications of this instrument, and those of the ordinary thermometer, the tension of vapour at the dew point is deduced(*a*) by the formula

$$f = f' - \cdot 0114\,(t - t')\,;$$

in which t denotes the temperature of the air, as shown by the ordinary thermometer; t' the temperature indicated by the hygrometer; and f' the tension of aqueous vapour at the temperature t'. When the temperature of the hygrometer is below the freezing point, or t' less than 32°, the numerical value of the coefficient is different, and the formula becomes

$$f = f' - \cdot 0102\,(t - t').$$

Anemometer and Rain-Gauge.—The direction and pressure of the wind, and the amount of rain fallen on a given horizontal surface, are now measured by the self-registering anemometer and rain-gauge of Mr. OSLER, the construction of which has been fully described by the inventor. The instrument is erected at the top of a building in the College, at some distance from the Observatory, and in a position in which the vane is freely exposed on all sides.

(*a*) For the proof of these formulæ see Dr. APJOHN'S Memoirs in the *Transactions of the Royal Irish Academy*, vol. xvii.; and the *Proceedings*, May 11, 1840. In the form given above the variations arising from the changes of atmospheric pressure have been neglected, as under ordinary circumstances inconsiderable; in order to take account of these variations, we must multiply the last term of each equation by $\frac{p-f}{30}$, p being the existing pressure in inches of mercury, as shown by the barometer.

There is also a rain-gauge, of the ordinary construction, on the roof of the Observatory itself; and the receiver of this being at a much lower level than that of the self-registering apparatus, a comparison of the results will serve to elucidate the question of the influence of height upon the quantity of rain received on a given surface.

H. LLOYD.

Trinity College, Dublin,
May 3, 1842.

EXPLANATION OF PLATES.

Plate I.

Fig. 1. Front elevation of Observatory.
Fig. 2. Plan of building, showing the disposition of the instruments.

Plate II.

Figs. 1. Stirrup of declinometer, front and side view. Full size.
Figs. 2 and 3. Lens-holder and scale-holder of collimator. Full size.
Figs. 4. Magnet furnished, and encompassed with copper ring. ¼th of size.
Fig. 5. Brass detorsion bar. ¼th of size.
Figs. 6 and 7. Suspension apparatus, side view and plan. Half size.

Plate III.

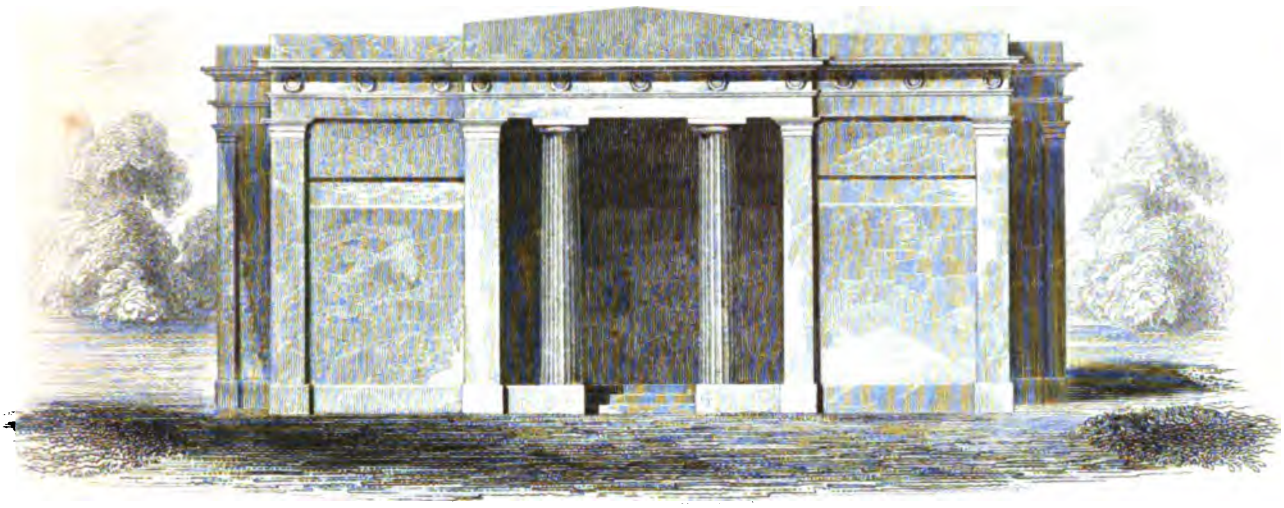
Figs. 1. Frame-work and box of declinometer, and bifilar magnetometer. ⅛th of size.
Fig. 2. Magnet of bifilar magnetometer, with stirrup and collimator. ¼th of size.
Figs. 3 and 4. Stirrup and wheel. Full size.
Fig. 5. Suspension apparatus. Half size.
Figs. 6 and 7. Diagrams.

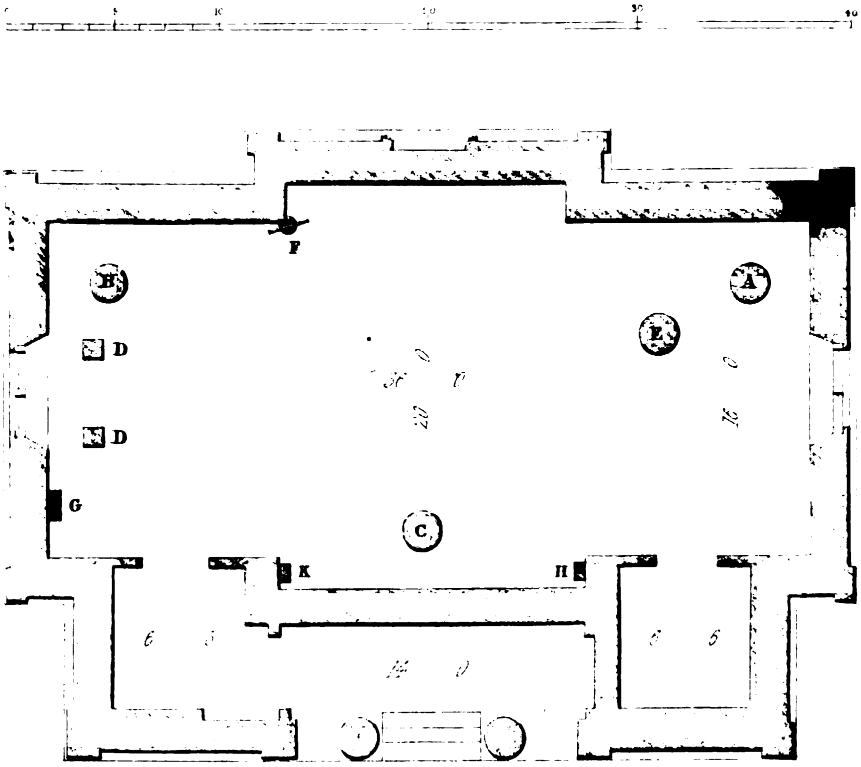
Plate IV.

Fig. 1. Balance magnetometer, elevation of instrument. ¼th of size.
Fig. 2. Plan of same.
Figs. 3 and 4. Magnet, seen from side and from above. ¼th of size.
Fig. 5. Brass bar. ¼th of size.

Plate V.

Fig. 1. Magnet of induction inclinometer, with stirrup and mirror. Full size.
Fig. 2. Induction inclinometer, elevation of instrument. ¼th of size.
Fig. 3. Plan of same.
Fig. 4. Reading telescope and scale. ¼th of size.
Fig. 5. Front view of scale. ¼th of size.





MAGNETICAL OBSERVATORY OF DUBLIN

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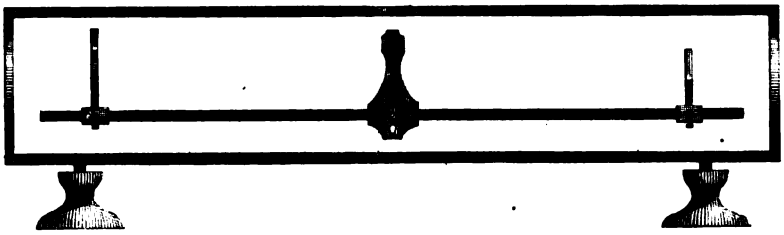

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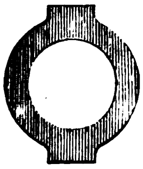

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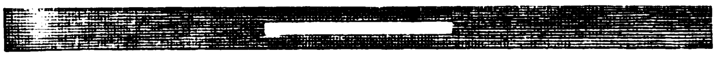

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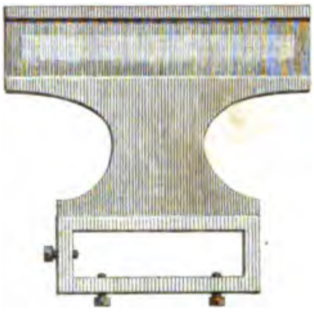

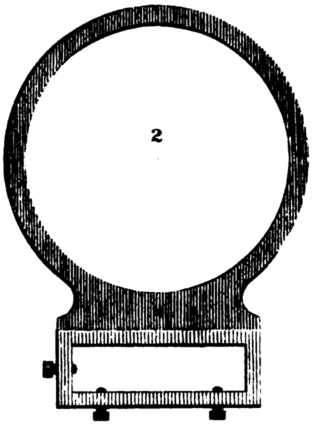

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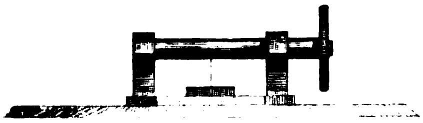

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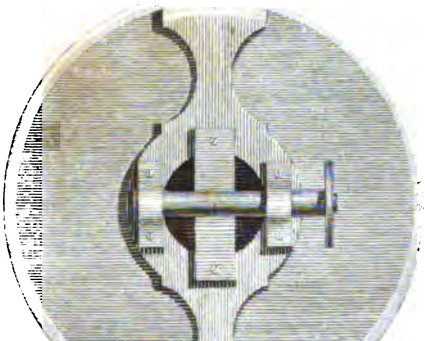

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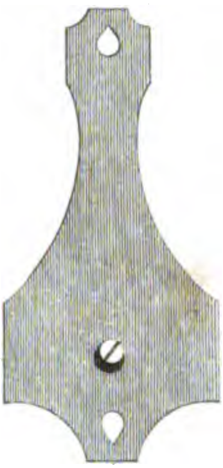

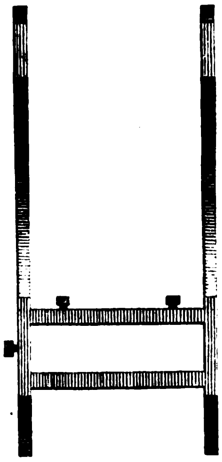

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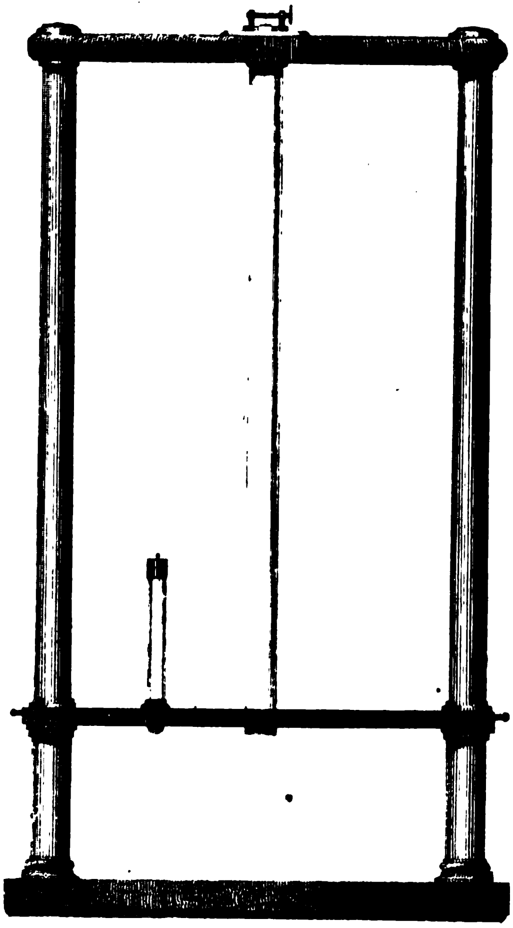

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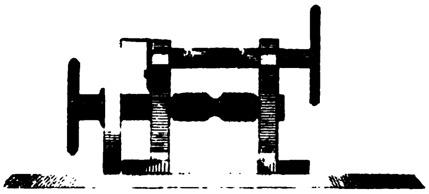

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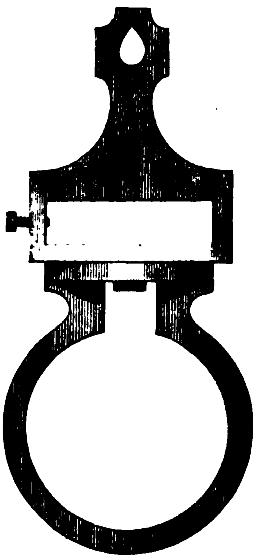

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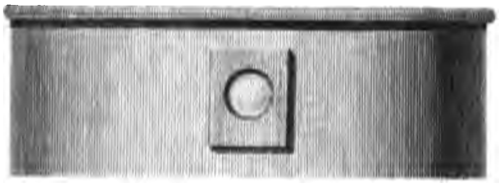

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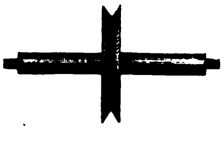

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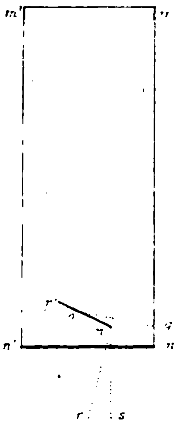

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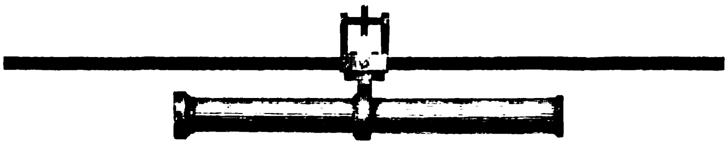

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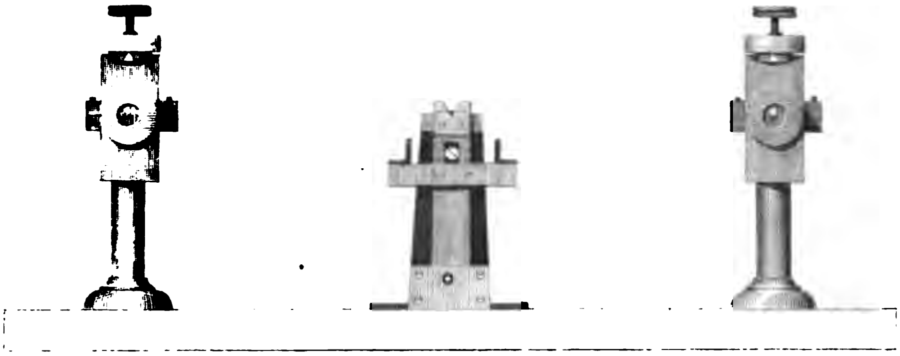

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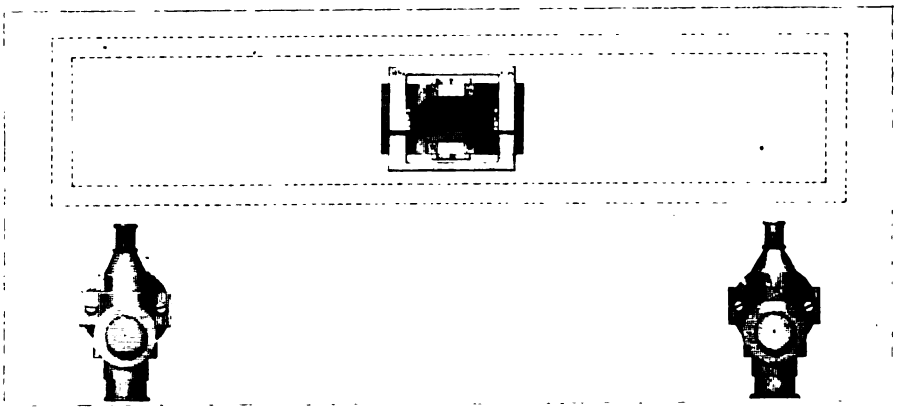

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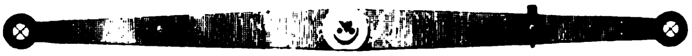

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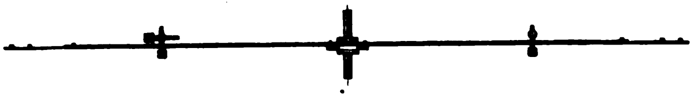

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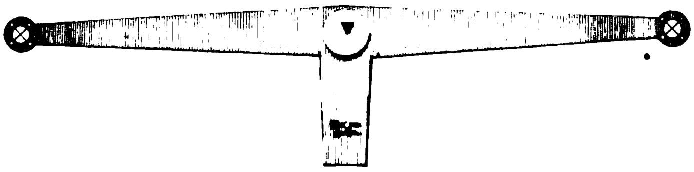

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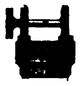

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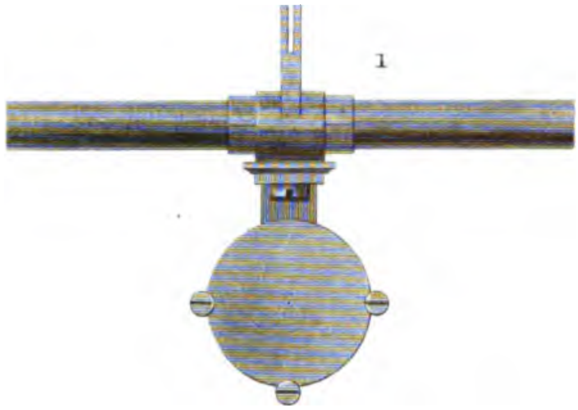

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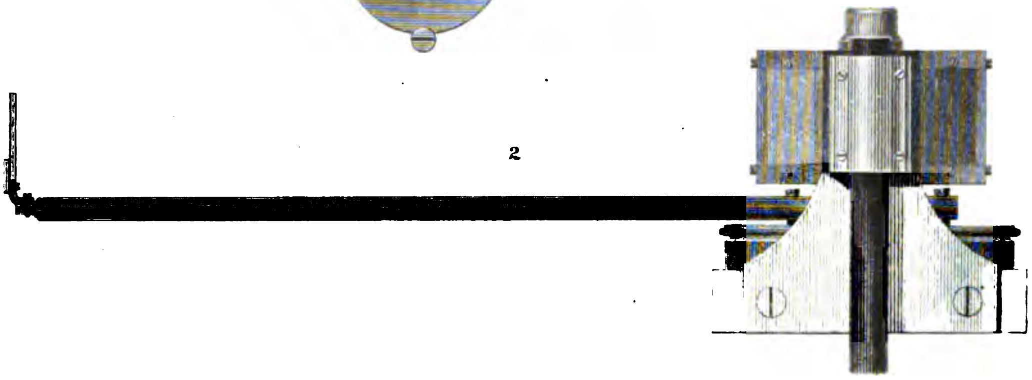

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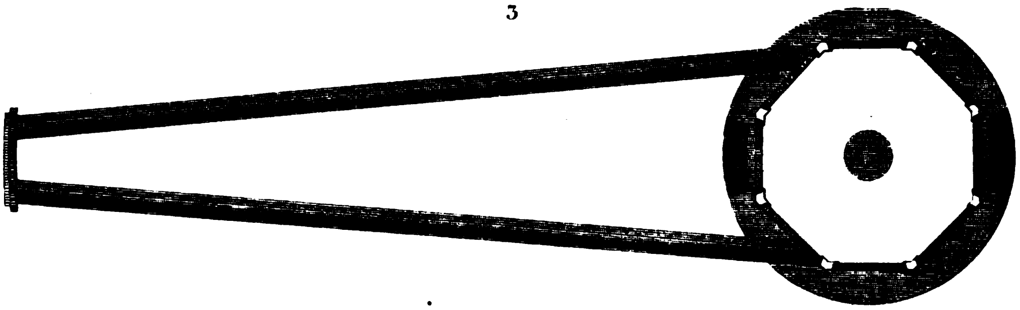

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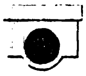

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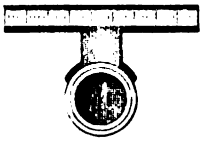

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